

# Teaching Mathematics: Discovering vs Storytelling

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## Abstract

There exist at least two approaches to the teaching of every subject, including mathematics. The first one is as old as the world (or, at least, as the education). According to this approach, to teach something means to outline what you teach. This approach is appreciated by the great majority of teachers. Because the realization of this approach in most cases reduces to the telling of stories about what is taught, I call this approach by «storytelling».

There exists also another approach, which goes from the famous statement by Aristotle: «We learn something only when we do what we learn». From this statement, I think, was born the following idea by Herbert Spencer: «What does it mean to teach? — It means to encourage systematically the students to their own discoveries» (as it was formulated by George Pólya and Gabor Segö in the epigraph to the preface of their book “Problems and Theorems in Analysis” and repeated in Polya, G., (1981)). This approach to teaching is natural to call «discovering».

In this paper, I intend to look closely to these approaches (mainly, with respect to mathematics), to identify their advantages and/or disadvantages, and to consider the work of brain hemispheres within these approaches.

**Keywords:** teaching mathematics, discovering of proofs, storytelling, functional brain asymmetry

## 1. Teaching Math as Discovering of Proofs

In mathematics, we have a tool that allows us

- to discover new,
- to confirm it, and
- to understand how and why are things going.

Such a remarkable tool, “three in one”! This tool is named proof.

Basing on this, the author has suggested in the year 2000 the so-called conception of humanitarian oriented mathematics teaching according to which

“to teach mathematics means to encourage systematically the students to discovering of their own proofs”.

Accordingly, in my teaching practice, I always try to teach my students to discover their own proofs. Let me illustrate this by one example. Consider the following

### Problem.

Peter once said: the day before yesterday I was 9 years old, and next year will turn 12. What is the date of Peter’s birthday, if what he said is true?

### Solution.

Let Peter said truth. Then next year Peter will turn 12. So, this year will be 11. So, in the past year, it turned 10. But the day before yesterday there were 9. So, 10 years was fulfilled yesterday. So, yesterday was the past year. And that can only be if yesterday was December 31. Thus, Peter’s birthday is December 31.

Look at the written text. It is proof. It is the proof that if Peter said truth than Peter's birthday is December 31. So, due to this proof we, first, discovered Peter's birthday, we, second, confirm our discovery, and we, third, understand why the things go exactly like this.

Note that the things go exactly like in this example with respect to every good proof. Good proof, as we already said, is "three in one": a tool for discovering, for confirmation, and for understanding.

## 2. Storytelling

But my dean (a physicist, not a mathematician) keeps telling me: "Why do you require proofs from your students? This leads to a low level of their estimates. Instead, tell them stories and ask them to retell you these stories, and you will see the growth of the estimates of your students".

Of course, I have no doubt that in the result of the proposed replacement the estimates of my students will increase, but I also clearly understand that this will be the estimates for different subjects. My estimates are for *mathematics* while new estimates will be for the *storytelling about mathematics*. I have to stress that most storytellers do not see the difference between the first and the second (one example of such storytellers the reader will see a bit later). I will explain this difference. In order to learn to swim isn't enough to listen to the stories *about swimming*. Everyone who wants to learn swimming should try *to swim*. Exactly these attempts will lead him to the fact that one happy moment he will suddenly *discover* inside of himself the ability to stay on the water. Without this *discovery*, he will not learn to swim and no storytelling about swimming can help him to make this discovery. Analogously, it is impossible to learn to ride a bicycle by listening to stories about riding a bicycle. It is impossible to learn to play the violin by listening to stories about playing the violin. And so on. But most teachers prefer not discovering, but storytelling.

### Example.

One of Ryazan's mathematics teachers invited me to his lesson. The lesson was about writing and reading number intervals. In the flow of the lesson, the teacher was in need to introduce the notation for infinite intervals, something like  $(1, \infty)$ . The teacher decided not merely to introduce the notation but also to tell the story about it. The story was as follows. "Children, scientists constantly meet with very large numbers, 100, 1000, 10000, 100000, 1000000.... But someday they met such a large number, such a large number (he repeated) that they did not even know how to call it. And then they decided to call it simply infinity."

I like very much this amusing story because it clearly shows what storytelling is.

{Story}-telling **IS** {Whatever-you-want}-telling!

Those readers who want right now object to me I ask to give a little patience. I am sure that they want to tell that I gave an example of a bad teacher and that with good teachers such things are impossible. Wait and see.

### Example.

Many times I read that Ancient Greeks instead of proofs produced drawings and wrote near them: "Look!" Someday I decided to verify if teachers I am working with know about this. My verification I divided into two questions. The first question was:

Do you *hear* that Ancient Greeks instead of proofs produced drawings and wrote near them: "Look!"?

The answer was "Yes". After that I asked the second question:

Do you *know* that Ancient Greeks instead of proof produced drawings and wrote near them: “Look!”?

The answer was: “Why do you ask it *again*? We already said you that we *hear* this!

Let’s draw from this preliminary conclusion: many people brought up on storytelling, mathematics teachers included, do not distinguish between ‘to hear’ and ‘to know’. I heard, so I know!

But this is, as I have already said, only preliminary conclusion. The main conclusion is much sadder. I repeated described above experiment many times, and always the result was the same. All the teachers I interviewed heard about this and even knew about it. Moreover, I read this on educational sites on the Internet and in dissertations on mathematics education, Ph.D., and Sc.D. Why did I call this phenomenon sad? **Because what was said about Ancient Greeks is simply impossible!** What is the main achievement of Ancient Greeks in mathematics? Introducing the idea a proof! And, further, proving the infinity of the set of primes, proving the Pythagoras theorem, proving the irrationality of square root of two, etc. How can inventors of proof omit proofs and write instead “Look!”? In no way! So did not the Greeks, but the Hindus! But over and over again I hear that Ancient Greeks...”.

This example shows clearly that

{Story}-telling **IS** {Somewhere-sometime-I-heard-something-like}-telling!

### **Example.**

When the perestroika (restructuring) began in the USSR, one historian told me with pride: “Now I can speak whatever I want”. I answered him that neither before nor after beginning the perestroika I could speak whatever I want; because being a mathematician I have to tell only what is proved, and what is proved always is true, so I am forced always speak only that what is true (not what I want!).

This last example shows clearly the key difference between the two approaches to mathematics teaching: teaching through discovering and teaching through storytelling. Under the first approach one forced to tell only that what is true while under the second approach one can tell whatever he wants. In particular, it shows that

{Story}-telling **IS** {Whatever-you-want-because-who-can-verify}-telling.

### **3. Storytellers about storytelling**

Above, the author has outlined his own opinion on storytelling. Now, turn to the opinions of storytellers themselves. We have chosen for this purpose, maybe, the most popular book about teaching mathematics through storytelling, *Teaching Mathematics as Storytelling* by Rina Zazkis and Peter Liljedahl. The book begins with the following claims.

“We like to tell stories. We tell stories about mathematics, about mathematicians, and about doing mathematics. We do this firstly because we enjoy it. We do it secondly because the students like it. And we do it thirdly because we believe that it is an effective instructional tool in the teaching of mathematics.”

What is this? The answer is evident; this is a storytelling about storytelling. The authors tell us what they want to see in storytelling. Let’s look closely at these statements. Let’s begin with the second statement,



They questioned me, how I can talk so disrespectfully about the storytelling if Steve Jobs himself gave to it such a high estimate.

First of all, I do not think that this estimate is high. Moreover, precisely in this form, this saying isn't true. In fact, isn't it absolutely clear that Jobs' statement is related not to all storytellers but to great storytellers only? After making this refinement, I ask my opponents, if they can confirm the Jobs' words. They were very surprised. "Why to confirm? This was said by Steve Jobs! Isn't it enough?" Before going further, let's mention the feature of the storytelling illuminated by this question, namely

{Story}-telling IS {Do-not-confirm-if-somebody-famous-tells-something-like}-telling.

Now, I can easily confirm the words of Steve Jobs. To do this, it is sufficient to show the following series of faces.



Who are these men? They are greatest storytellers of the past two centuries. Look at these faces and repeat word by word the statement by Steve Jobs. You will see that these words constitute extremely precise characterization of these creatures. In fact, they were most powerful persons in the world in their times. And they set the vision, values, and agenda of the entire generations.

Tragic consequences of their activities are well known. It is very important to understand why did this happen. In my opinion, the main reason for this consists in that the human beings ... aren't parrots!

In fact, what can do parrot? He can listen, memorize, and repeat. Maybe, he can even drink your cocktail instead of you.



But there is such a thing that he will not do under any circumstances: **the parrot will not believe you!** This is absolutely excluded for him.

Now, let us imagine that human beings would be as smart as parrots. What would they do with the *Communist Manifesto*? Read, memorize, and repeat. Well, maybe also drink cocktail. And that is all! But human beings are not parrots. They are much more stupid. **They need to believe!** Even if no one forces them to do this! Why, for what — it does not matter! Believe just so! They readily believe anything, even the most ridiculous nonsense! As Tertullian said: I believe, for it is absurd!

So, they believed what is written in the *Manifesto*. But the belief is a very dangerous thing. For the belief, the human is ready to give his life and take the life of another. Exactly this has happened. Those who believed the sinister tales of Marx rose up against those who did not want to believe and the rivers of blood flowed through the Earth.

Let us note this feature of the human beings: they need to believe. Let's say the same thing in more rigid words: human beings suffer from the severe form of a dangerous disease, unknown to the representatives of other species. This disease is called abnormally high suggestibility.

That was noticed long ago. For instance, in *The Port-Royal Logic* is written:

“[T]here are no absurdities too groundless to find supporters. Whoever determines to deceive the world may be sure of finding people who are willing enough to be deceived, and the most absurd follies always find minds to which they are adapted.”

(Arnauld and Nicole, *The Port-Royal Logic*, Discourse I)

And in the twentieth century, Johan Huizinga wrote in his book-warning *In the Shadow of Tomorrow*:

“The abnormally high suggestibility is that Achilles' heel, to which the advertisement beats the modern man. This applies to both commercial and political advertisement. And all this leads to the loss of critical judgment. In the result, huge masses of people are subject to the thoughtless execution of what is contained in the thoughts and phrases that are imposed from the outside.”

And Huizinga warned that this could lead Europe to disaster. (He was talking about Europe, but his prediction came true for the whole world. As is said in one of Murphy's laws, “Things are not going so badly. They go much worse”.) But who wants to listen to thinkers? They like listening to storytellers with their beautiful fairy tales, and not predictors of collapse. So, nobody listened to Huizinga, but as a result ... The result was already mentioned above and is well known. And we must understand that this shows us what storytelling is! So, we see that we need to augment the words by Steve Jobs as follows (Steve Jobs' words are printed in blue, mine in red):

**WARNING!**  
**People, be careful!**  
**The most powerful person in the world**  
**is the storyteller.**  
**You will not even notice how**  
**the storyteller sets the vision,**  
**values, and agenda**  
**of an entire generation that is to come.**  
**This can have**  
**and throughout the history of mankind**  
**many times really had**  
**the most tragic consequences.**  
Steve Jobs and Aslanbek Naziev

Adherents of the storytelling may object that they have not in mind something bad. To this, I can ask, what bad have in mind creators of *The Communist Manifesto*? “A spectre is haunting Europe – the spectre of communism”. What’s dangerous? Well, spectre. Well, is haunting. Well, Europe. What’s dangerous? Especially for Cambodia in the twentieth century. But when this spectre got to Cambodia in the 1970’s, the expression “the rivers of blood” became literal. Of the seven million people, Cambodia lost three million in the name of the triumph of Marx’s fairy tales.

Those who take the problems of education seriously must understand that the storytelling exploits and aggravates the most dangerous features of the human race, excessive credulity, and abnormally high suggestibility. Without this features storytelling would be impossible. In order to understand this, it is enough to imagine how at the beginning of storytelling listeners tell to the storyteller, “You can tell whatever you want but you must know that we will not believe you”. What would the result be of this declaration? Storytelling would become impossible!

This thought experiment shows that what is ordinarily called storytelling is in fact not merely story-telling but also story-believing.

### **5. Two Hemispheres of the Brain**

Does all that was said above mean that storytelling is useless? No, I do not think so. It is useful in its own time and place. In order to pick out these time and place, we need tell some words about the work of the brain’s hemispheres.

The researcher of processes of intelligence and creativity Paul Torrance (1915–2003) conducted an experiment in the course of which he established four types of thinking:

- left hemisphere thinking as one that is built on logic and analysis;
- right hemisphere thinking where the thinking process is directed by emotions, intuition, and images;
- mixed thinking where equally active are both right and left hemispheres, each of which is switched on at the right moment;
- integrated thinking when the right hemisphere and left hemisphere thinking work simultaneously.



Torrance stressed that among these types of thinking there are no good or bad types: everyone has advantages and disadvantages. However, the scientists and educators in recent years have increasingly attached importance to the development of right hemisphere thinking. In this connection let’s recall the following words by outstanding Russian neurolinguist and neuropsychologist Vyacheslav Vsevolodovitch Ivanov:

“The right hemisphere in the norm (when censorship of the left hemisphere is carried out) is wordless, and this is the source of its creative potential.”  
(Ivanov Vyach.Vs. 1929–2017)



Let us stress that to uncover the creative potential of the right hemisphere, censorship of the left hemisphere is necessary. Now note that this censorship isn't inborn. It has its own time and place. It is established in the early childhood in the course of the mother's storytelling. In the first years of life, the child and mother establish special emotional relationships that are characterized by the child's unlimited trust in the mother. When the mother points a finger at the dog and says in a special voice to the child that it is a doggie, the child does not just remember the name, he *believes* that it is a doggie. Here, this credulity is in its proper place and is vital. (Imagine, what would be if the child constantly questioned the mother's words!) But what is appropriate in early childhood gradually becomes inappropriate and even dangerous as the child grows up. The infamous phenomenon of child theft is based, precisely, on the child's overconfidence to outsiders, and this trustfulness needs to be specially disaccustomed.

So, early childhood is the proper place for the storytelling. At later stages of development, it must give way to more advanced tools, retaining the role of only a minor auxiliary tool.

#### **6. Discovering against uncontrollable believing**

One of those more advanced tools mentioned at the end of the previous section pointed out Rudolf Steiner in his *The Philosophy of Freedom*.

“We no longer want *to believe*; we want *to know*. Belief demands the acceptance of truths which we do not wholly comprehend. But the individuality which seeks to experience everything in the depths of its own being is repelled by what it cannot understand. Only that knowledge will satisfy us which springs from the inner life of the personality, and submits itself to no external norm.” (Rudolf Steiner, *The Philosophy of Freedom*)



Now, it is natural to ask, “What does it mean ‘to know’?”

It was explained by Aristotle:

“‘To know’ means ‘to discover with the help of proof’.”

So, we are returning to the very beginning of our paper: to the discovering of proofs. This is the most effective counter-sanction against storytelling in adulthood.

Wherein, every time, passing to the storytelling, the teacher should warn the students, “Now we leave the safe region of the discovering and turn to the extremely dangerous zone of storytelling. Be careful! Do not take a single my word to faith! Check and double-check, check and double-check! And not because I’m so ugly, but because so ugly is storytelling!”

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### Aslanbek Naziev’s Biodata:



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**Teaching experience:** Developed and implemented in the practice of teaching at university level about ten author's courses in mathematics and mathematics teaching; in particular, “A Course in Foundations of Mathematics” for students of Pedagogical Universities (4 semesters); Prepared ten Ph.D. students successfully defended their thesis on the methodology of mathematics teaching.