Controversies about the Introduction of Non-Classical Logics

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Abstract

Logic is a set of well-formed formulae, along with an inference relation. But the Classical Logic is bivalent; for this reason, very limited to solve problems with uncertainty on the data. It is well-known that Artificial Intelligence requires Logic. Because its Classical version shows too many insufficiencies, it is very necessary to introduce more sophisticated tools, as may be Non-Classical Logics; amongst them, Fuzzy Logic, Modal Logic, Non-Monotonic Logic, Para-consistent Logic, and so on. All them in the same line: against the dogmatism and the dualistic vision of the world: absolutely true vs. absolutely false, black vs. white, good or bad by nature, Yes vs. No, 0 vs. 1, Full vs. Empty, etc. We attempt to analyze here some of these very interesting Classical and modern Non-Classical Logics.

Keywords: Mathematics; Classical or Formal Logic; Non-Classical Logics; Artificial Intelligence.

1. Introduction

We may think that Fuzzy Logic would be a recent field, in which people have been working for a very little time. But it surely originates from the Old Greek philosophers and scientists, in particular, from Plato (428-347 B.C.), and his reverenced disciple, Aristotle (384-322 B.C.).

From such initial thinkers we may mention two essential names: David Hume (1711-1776), and Immanuel Kant (1724-1804). They have managed to obtain a detailed analysis of such concepts. Both philosophers conclude that both reasoning and knowledge are reached through life experience.

In fact, David Hume has chosen the so-called Common-Sense Logic. Kant possesses the opinion that only mathematicians can provide very clear definitions, detecting some inner contradictory principles on Classic, named also Formal or Mathematical Logic. Thus, in the beginning of the 20th century, the British mathematician Bertrand Russell (1872-1970) spread the idea that the Mathematical Logic reveals inherent contradictions. He produced an extensive study on the vagueness of the language, concluding that the vagueness is a degree. Georg Cantor (1845-1918) and his crisp or classic set theory should also be mentioned. In some way, he may be considered the precursor of both Zadeh´s fuzzy set theory, and of the Uncertainty Theory of Charles Sanders Pierce (1839-1914).

The Russian logician, symbolist poet and thinker, Nicolai Alexandrovich Vasiliev (1880-1940), must be remembered, because it was a notable forerunner of Multi-Valued Logic, and in particular, of Para-consistent Logic. N.A. Vasiliev presented a lecture (on 1910) entitled “On Partial Judgements, on the Triangle of Opposites, on the Law of Excluded Middle”, where for first time appears the idea of Non-Aristotelian Logic, without the Excluded Middle and Non-Contradiction Principle. In homage to the “imaginary geometry” of N. Lobatchevsky, N.A. Vasiliev baptizes the new construct as “imaginary logic”. He supposed that this logic may be valid for the possible worlds where the Aristotelian laws did not hold. He also introduces the concept of “metalogic”. These ideas fall in a long oblivion, being recovered in the early1960s as the basis of the Para-consistent Logic. Therefore, due to his work, he must be considered as a precursor of MVL.

The Austrian thinker Ludwig Wittgenstein (1889-1951) analyzed the different meanings (senses) of a word. This way he observed that, in a language, the same word can express different moods and manners.

Furthermore, the Austrian logician and mathematician Kurt Gödel (1906-1978) showed in 1932 that Intuitionistic Logic is not finitely-many valued. This is why he defines an intermediate phase between Intuitionistic and Classical Logic: the so-called Intermediate Logic.

The Polish logician Jan Lukasiewicz (1878-1955) developed the first Vagueness Logic; in fact, a three-valued logic, with three values, 0, ½ and 1, corresponding to False, Possible and Truth. Afterwards he generalizes to four-valued logic, and many-valued logic (MVL, by acronym). Therefore, he introduces the membership degrees of each element to each corresponding set. Thus, we dispose of a membership function of which range covers the unit interval of the real line, $I = [0, 1] \subset R$.

The father of the new formalization of the term "fuzzy", jointly with the Fuzzy Logic and Fuzzy Set associated theories, was Lofti Asker Zadeh (1921-), who, in 1965, published his famous and seminal paper, the so-called "Fuzzy Sets" (Many-dimensional modal logics: theory and applications, 2003). Such ideas collect the precedent work of many authors of different disciplines.

Among them must be mentioned Max Black (1909-1988), philosopher and also quantum physicist, who in 1937 proposed the necessity of a theory of "vague sets". Also the imminent British mathematician Bertrand Russell (1872-1970), considered a great precursor, who dedicated his work to searching logical contradictions and their plausible solutions.

The German quantum physicist Werner Heisenberg, with his famous "Uncertainty Principle", related in a way with the above theories; at least increasing in the same ambient. Moreover, the German logician and physicist Hans Reichenbach (1891-1953), founder of the Berlin Circle, and great philosopher about probability, logic, space-time, etc., has also analyzed these new concepts.

It must also be mentioned the notable contribution of the Polish mathematician Jan Lukasiewicz (1878-1955), who together with Alfred Tarski (1902-1983) has formulated a logic of n truth values, being $n \geq 2$, clarifying many important technical questions through the initial introduction of a third value, "possible". The same as all those Polish mathematicians was Emil L. Post (1897-1954), another supporter of additional truth degrees.

All those elements contributed to produce the magma from which derive the famous papers of Lofti A. Zadeh (not only the aforementioned "Fuzzy Sets" (1965), but also the ´Fuzzy Algorithm´, some years afterwards).

At the beginning (as almost all the new ideas) the papers of Lofti A. Zadeh did not obtain very good scientific reception, but with the past of the time they have gained more and more people, So such theories were considerably extended, and accepted by very professional scholars. The initial resistance against the Fuzzy Logic may be very surprising, but certainly when Zadeh visited IBM to expose his ideas, they said that they are not interested! It is, in fact, a memorable example of commercial occidental vision.

The purpose, or objective, of Lofti A. Zadeh was to create a formalism to manage efficiently the natural imprecision of human reasoning. In 1971 appears his paper called "Quantitative Fuzzy Semantics", where appear many formal elements of Fuzzy Logic, and their methodological aspects, with derivations that until today are frequently used on applications. From 1973, with the basic theory of Lofti A. Zadeh on fuzzy controllers, many researchers initiate the application of Fuzzy Logic to control diverse processes.

2. Non-monotonic Logic

We call *Non-monotonic logic* a logical system where the logical consequence relation is non-monotonic.

Many of the logical systems, as for instance Mathematics, have a logical consequence relation which are monotonic. Because adding some formula to a theory never produces a reduction of the set of consequences.

We may note a crucial point: we do not modify here our disposable base of affirmations or base of facts (by AI terminology), because if we do this, the possibility of modifying the set of consequences would be evident. Instead, our argumentation departs to maintain the same elements on the original knowledge base, only now enlarged with new propositions, or new information. This occurs very frequently in Medicine, where the new discoveries continuously invalid old beliefs. So, as the knowledge advances, what today is prescribed, tomorrow will be forbidden. And this produces the necessity of quickly going out of the Aristotelian or Formal Logic, where the reasoning is of monotonic type, as in Mathematics. This description shows the mechanism of the non-monotonic reasoning, with a totally provisional and ever revisable character, evolving in time; the opposite of Mathematics.

3. First Approaches to Fuzzy Logic

There are at least two approaches to Fuzzy Logic. The first one may be considered related to Multi-Valued Logical tradition; properly, the Peter Hájek´s school, around the Charles University of Prague. The procedure must be to fix a set of designed values, and then, to define an entailment relation. We may define a suitable set of axioms and Inference Rules, as engine for its deductive apparatus.

The second line of advance would be headed by Pavelka, Goguen, and so on, being directed on to provide a deductive apparatus in which approximate reasoning may be admitted. It will be reached by a suitable fuzzy subset of logical Axioms and Fuzzy Inference Rules.

Between both approaches, it will be very different the logical consequence operator. In the former case, it gives the set of logical consequences of a given set of Axioms, whereas in the latter case it gives the fuzzy subset of consequences of a fuzzy subset of hypotheses.

Aristotle established their Laws of Thought:

- Identity Principle
- Principle of Contradiction: not P and not P.
- Law of Excluded Middle: either P or not P.

They have conditioned -and in some cases obstructed- the development of Logic in Western culture. This fact is less oppressive in Oriental countries (China, Japan, Korea, India, etc.), because the Buddhist tradition admits with naturalness, on the things and thoughts, the possibility of simultaneously be and not to be, and that the propositions may be both, true and not true by the assignation of a gradation. This connects directly with the Fuzzy Logic and the Theory of Fuzzy Sets, who introduced the membership function, which range to move into the unitary real interval, denoted by

$$I = [0, 1] \subset R$$

In Ancient Greek some philosophers have pronounced affirmations that are compatible with a new way of think. So Plato in their Dialogues makes a good compilation of thoughts of Socrates, his teacher, and other precedent thinkers. As aforementioned, and according to Aristotles, Heraclitus of Abdera and Anaxagoras both denied the Non-Contradiction principle, according to it is not possible to reach simultaneously p and no p. Also may be interesting to consult Protagoras about this Principle.

The psychological version of such Principle of Non-contradiction may be this: Nobody can to think “p” and “no p”.

Another properties that characterizes the Classical Logic may be: Monotonicity of Entailment, Idempotency, Commutativity of Conjunction, and also Double Negative Elimination, in this case by NOT (NOT A) = A.

Also are characterized these formal logics so-called Classical Logics by the Laws of De Morgan,or De Morgan Duality, according to

NOT (P AND Q) = (NOT P) OR (NOT Q)
NOT (P OR Q) = (NOT P) AND (NOT Q)

I. e., every logical operator is dual to another.

Fuzzy Logic, as infinite-valued logical system, rejects the Excluded Middle Principle, admitting as truth-values any real number that belongs to the unit real interval, I = [0, 1].

If expressed in modern formulation, the collection of Fuzzy Sets, with the operations of union, intersection and path to the complementary defined by Zadeh, will be not a Boolean Algebra. So, it is very different to the now classical theory of George Cantor, about the Crisp Sets. And it is because the violation of both principles, Non-contradiction and Third Excluded.

4. Oriental Thought

Searching on the old Oriental doctrines, we can found some tendencies of Buddhism very related with the degrees of truth, and also the two-truths doctrine.

Nāgārjuna (150-250 A. C.) was the Indian founder of the Madhyamaka school of the Buddhism Mahayana. The term Madhyamaka signifies "Middle Way, or Via Media". It was the more prominent philosopher on the Indian tradition.

Nāgārjuna was also a practitioner of traditional Indian Ayurvedic medicine. Between his conceptualizations, we can found some descriptions of the circulatory system and blood tissue. In Western medicine, at least until now, Ayurvedic medicine is considered alternative, as merely a complementary treatment, but never replacing the traditional diagnosis and "farmacopea" of Western expert. Balance is emphasized; suppressing natural urges is seen to be unhealthy, and doing so may almost certainly lead to illness. So, the people are cautioned to stay within the limits of reasonable balance and measure.

His primary contribution is in the use of the concept of "emptiness" which brings together other key Buddhist doctrines, to refute the metaphysics.

For Nāgārjuna, as for the Buddha in the early texts, it is not merely sentient beings that are "selfless" or non-substantial; all phenomena are without any "self-nature", and thus without any underlying essence. They are empty of being independently existent. This is so because all things arise always dependently: not by their own power, but by depending on conditions leading to their coming into existence, as opposed to being.

Nāgārjuna was also instrumental in the development of the two-truth doctrine, which claims that there are two levels of truth, one which is directly true, and one which is only conventionally or instrumentally true. So, "this world is supported by a polarity, that of existence and non-existence… Everything exists: That is one extreme. Everything doesn't exist: That is a second extreme. Avoiding these two extremes, the Tathagata teaches the Dhamma via the middle..."

Nāgārjuna differentiates between conventionally true and ultimately true teachings, but he never declares any conceptually formulated doctrines to fall in this latter category. This was famously rendered in his Tetralemma, with the logical propositions:

- X (affirmation)
- non-X (negation)
- X and non-X (both)
- neither X nor non-X (neither)

So, we may establish three original traditions of Logic: Greek, Chinese and Indian traditions. This latter continued until very modern times, through the Navya-Nyaya school of logic, because the Rigveda contains some specuculaive considerations in terms of logical divisions. More exactly,

- "A",
- "not A",
- "A and not A", and
- "not A and not not A",

which coincides with the above Tetralemma.

Panini, in the 5th. century B.C., developed a logic that have certain interesting similarities with Boolean logic.

The Navya-Nyaya school developed a conceptual scheme that allowed solve problems in Logic, by developing theories very similar to Gottlob Frege´s ideas.

All these schools are discovered by British scholars in the late 18th century. We mention the H. T. Colebroke´s analysis of inference, by comparison with Aristotelian logic, resulting that the Aristotelian syllogism could not account for the Indian syllogism.

The German mathematician Hermann Weyl (1885-1955), in 1924, wrote that "Occident mathematics has in the past centuries broken away from the Greek view, and followed a course which seems to have originated in India, and which has been transmitted, with additions, to us by the Arabs; in it the concept of number appears as logically prior to the concepts of geometry".

In fact, Indian logic has deeply influenced on many of modern logicians, as Charles Babbage, George Boole, or Augustus De Morgan. They belong to 19th. Century, but may be named "modern" in many senses. The same De Morgan says that "the two races which have founded the Mathematics, those of the Sanscrit and Greek languages, have been the two which have independently formed systems of logic".

Therefore, mathematicians are finally aware of the notable influence of Indian Mathematics/Logic on the European Mathematics/Logic.

5. Rule-Based Systems

The rules show a great advantage on the Classical Logic. Usually, the rules must appear grouped, as a system of Rules, each of them with its antecedent and consequent. They have two procedures of firing, according which we apply chaining forward or chaining forward. It is because in each system we can proceed from the respective antecedents to their consequents, or vice versa, until reaching the goal fact (or "concepto-meta", in Spanish), i.e. the desired last fact.

Logics are systems intended to codify Rules for preserving some semantic property of propositions across transformations. In particular, Multi-Valued Logics are intended to preserve the property of being designated (designationhood).

In the Classical Logic the Reasoning was Monotonic, with inferences without contradiction with the pre-existing, in SBR. Nevertheless in the Rule-Based Systems (RBS, by acronym), we may delete facts or affirmations of the Base of Facts, according the new inferences. This makes the Reasoning Non-Monotonic, because we can modify the conclusion. Then, appears a crucial question: what should we make with the conclusion of the affirmation now invalided?

For this problem, we need to introduce the concept of Type of Dependence of a Rule, which can be Reversible, if we delete the affirmations, then we automatically delete the above inferred facts; or Irreversible, if the facts, once inferred, will be not deleted, neither changed. And in the case of some applicable rules at time, which one must be firstly executed? Such Rules constitute, in each step, the Conflict Set (obviously, a dynamic set). The subjacent decision problem is called Resolution of Conflicts or Control of Reasoning.

Rules appear usually as Rule Systems, in the form:

IF variable IS property THEN action.

So, for instance, a temperature regulator may use a fan which runs according to this Rule-System:

- IF temperature IS very cold, THEN stop fan.
- IF temperature IS cold, THEN turn down fan.
- IF temperature IS normal, THEN maintain level.
- IF temperature IS hot, THEN speed up fan.

Note that there is no "ELSE", because all the Rules must be evaluated. And that the temperature might be either cold and normal, or normal and hot to different degrees.

The typical operators of Boolean Logic ($\cup$, $\cap$, and c) remain valid in Fuzzy Logic, being implemented through the Maximum, Minimum, and Complement.

6. Non-Classical Logics

The essential difference between Classical and Non-Classical Logics resides on the condition of bivalence. Because the Classical Logics were bivalent, or two-valued. This means that

they are understood as dividing propositions into True and False ones. Whereas Non-Classical Logics are those systems which reject the bivalence condition because they admits another intermediate truth values. In fact, systems as Fuzzy Logic have an infinite number of truth degrees, being represented by a real number between 0 (absolutely false) and 1 (absolutely true).

Intuitionistic Logic was proposed by the Dutch mathematician Luitzen Egbertus Jan Brouwer (1881-1966). It was introduced as the correct logic reasoning on Mathematics. The key point was his rejection of the Excluded Middle Law, the Double Negative Elimination, and De Morgan´s Laws as part of his theory. His also Dutch student, Arendt Heyting (1898-1980) studied such Logic formally, because Brouwer was opposed to any formalization of his Logic. The interest of Intuitionist Logic is very considerable in Computer Science, because it is a constructive logic, therefore between logics of what computers can do.

Linear Logic also rejects idempotency of entailment. It disallows arbitrary contraction in addition to arbitrary weakening.

Weakening -many times called thinning- is an Inference Rule, which permits for such systems a monotonic entailment, just in case this Rule is admissible. In most Logics, Weakening is either a Meta-theorem or an Inference Rule, if the logic does not present an explicit rule.

When a logical system possesses this property, it is called Monotonic Logics, in order to differentiate from Non-Monotonic Logics.

Computability Logic is a semantically constructed theory of computability, being opposed to Classical Logic, which in fact will be a theory of truth. Computabilty Logic extends and integrates different logics, between them, linear, intuitionistic, and classical logics.

Reasoning about Knowledge may also be referred to in cases such as those when statements specifying that something is not known needs to be retracted when that new knowledge is learned.

Relevant or Strict Logic is encountered when every Hypothesis is necessary for the conclusion.

Paraconsistent Logic rejects the Non-Contradiction Principle. It is so both in Relevant Logic, and in Dialetheism.

Abductive Reasoning, if we can derive the most likely explanations departing from the known facts.

Linear, Relevant, and Non-Monotonic Logics all them rejects the Non-Contradiction Principle.

Modal Logic introduces two new modes of truth: Necessity (denoted by □), and Possibility (denoted by ◊). But it is not truth conditional. For this reason, it has been proposed as a Non-Classical Logic. Nevertheless, it is frequently formalized with the Excluded Middle Principle, being bivalent its relational semantic. Therefore, such adscription to non-classicism may be disputed.

A formal Modal Logic represents modalities using modal operators. Such modalities or modes may be of one of these three types: Necessity, Possibility, and Probability.

Here follow some historical aspects of Modal Logic. In the works of Aristotle some passages may be considered as anticipations of Modal Logic, showing us their connections with time and potentialities.

In Medieval Scholastics, we can find other examples of reasoning in a modal manner: for instance in the works of William of Ockham, and John Duns Scotus, analyzing concepts of essence and accident.

The modern Modal Logic was founded by Clarence Irving Lewis, who in his thesis and articles, from 1910, established a new way, culminating in his book entitled Symbolic Logic (1932). It develops the models designed by (S1)-(S5). In more recent times, Samuel Kripke began developing its Modal Semantics, in 1959. It is the now-standard Kripke´s Semantics, where it appears the terminology of "possible worlds" semantics. The simplest model of modal logic is denoted by K, in reference to this author (Kripke).

So, Modal Logic signifies an extension of Classical Logic with the non-truth functional operators, also called "modal" operators. But according to many authors, it will be considered outside of the class of Non-Classical Logics.

7. Deviant and Extended Logics

Bertrand Russell said that "there are two kinds of science: the old, which is the official, and a new science, that most of the old look with horror. The result is a constant battle between the old minds who admire science for its older and younger men, who appreciate the value of the work of their peers. To some extent, this struggle is useful, but beyond a certain point, it becomes disastrous" [Autobiography, p. 722, 2010]. This lucid commentary typical of the great mathematician and philosopher, seems tailored to the controversy that we study, as has rarely been given in the history of the new scientific theories such bitter and unjust insults as those that occurred with the development and deployment of Non-Classical Logics, and particularly of Fuzzy Logic, Fuzzy Logic or also called Heuristic Logic. The logic is a well-constructed set of formulas, with a ratio of inference. However, classical logic is bivalent; therefore, it is very limited when it comes to solving problems with uncertainty in the data, which are most common in the real world. It is well known what artificial intelligence logic requires. Because its classic shows too many shortcomings, it is necessary to introduce more sophisticated tools, such as non-classical logics, including fuzzy logic, modal logic, nonmonotonic logic, the order-consistent, and so successively. All in the same line: against dogmatism and dualistic view of the world: absolutely true vs. false absolutely, black against white, good or bad by nature, Yes vs No, 0 vs 1, full vs. empty, and so on. We try to analyze here some of them, both classical and modern non-classical logics being very interesting, focusing in particular on the theme of your reception, which was quite different between countries.

At first (like almost all new ideas), the work of Lotfi A. Zadeh and other precursors, such as Max Black (1909-1988) and Jan Lukasiewicz (1878-1956), had no scientific enthusiastic reception, but over time, these ideas gradually gained acceptance in many cases, with the emergence of inspired followers. So these theories have grown considerably in a way, being accepted by scientists and today's most professionals.

Initial resistance against fuzzy logic can be very surprising, but certainly when Lotfi A. Zadeh visited IBM to present their ideas, the executives said their thoughts were not interested. This fiasco is, in fact, a more memorable example of the usual intellectual blindness to the new, very common in some Western companies.

Non-classical logics have been meeting with varying degrees of acceptance, but it is fuzzy logic which has found more resistance to being accepted in many academic circles. The purpose of Lotfi A. Zadeh was the creation of a formalism to deal more effectively with uncertainty and imprecision nature of human thought. The essential difference between classical and nonclassical logic is in the treatment of multi-valence. Since classical logic is bivalent, or two-valued. This means understanding the division of propositions between propositions propositions totally true and totally false. While non-classical logics are systems that reject the condition of ambivalence, supporting other intermediate truth values. In fact, fuzzy logic systems have an infinite number of degrees of truth, being represented by a real number between 0 (completely false) and 1 (completely true). Hence they are called Multi-Valued and Multi-Valued Logic as the "Fuzzy Logic".

Intuitionistic logic was proposed by the Dutch mathematician Jan Luitzen Egbertus Brouwer (1881-1966). He introduced himself as the quintessential correct logical reasoning in mathematics. His key point was the rejection of the Law of Excluded Middle (or excluded middle), the elimination of double negation, and the presence of Augustus De Morgan laws, as part of his theory. Your student-Dutch-Arendt also Heyting (1898-1980) studied the formal logic, as L. E. J. Brouwer was opposed to any formalization of logic. The interest of intuitionistic logic is very significant in Computer Science because it is a constructive logic, therefore, it lies between the logic of what computers can do.

Meanwhile, Modal Logic introduces two new ways of truth, of Necessity and Possibility. It has been proposed as a non-classical logic. However, often formalized with the principle of excluded middle, and relational semantics is bivalent her. Therefore, the ascription to the "classical"

cannot be in doubt. But you can consider a modal formal logic, representing patterns of use of modal operators. These modes or forms may be: Necessity, Possibility and Probability.

8. Controversies against Fuzzy Logic

One of the sharpest and persistent controversies was promoted by the "patriarch" of the probabilistic Italian academy, Bruno de Finetti (1906-1985), and his school. The conclusion we reach is that fuzzy logic and probability are two different ways of expressing uncertainty. Because, while the probability theory can be used to represent subjective belief in the theory of fuzzy sets, using the concept of belonging to a fuzzy set, a variable extent verifies the property that defines a set. In probability theory uses the concept of subjective probability. For example, how likely is that what I think is a given variable within a set? Although this distinction is mainly of philosophical nature, the possibility measure which gives us fuzzy logic is intrinsically different from the probability measure "classic" and, therefore, not directly equivalent. However, many statisticians were persuaded-and still are, by the influential work of the author before mentioned about only one type of treatment of uncertainty, for a mathematical point of view is needed, and therefore the fuzzy logic would be superfluous. It's kind of jealous rage after finding that a potentially strong rival invades our "little plot". Moreover, according to Bart Kosko (1960-), the probability is a sub-theory of fuzzy logic, as probability only handles one kind of uncertainty. Thus, Lotfi A. Zadeh (1921-) argues that fuzzy logic is different in character from probability, and not a substitute for it.

Let's look at our research, not only at the characteristics of each one of these non-classical logics, their interrelationships and possible applications to various fields, but also and especially at the lines of resistance observed for its implementation and use in our universities, to better understand its development and progress. That is, Controversies and Disputes have arisen before its appearance, as a remarkable example of the questions that often accompany any new scientific theories. But even so, in this particular case have shown a fiercer fury, especially in Western countries, instead being very well accepted, and naturally, in Asian countries, especially in Japan. Has the typical mental blindness, some inertia opposed to change, or has been a problem of cultural tradition, imbued as we are a strong Aristotelian tradition in our school systems...? Are all of them different issues of great interest for the advancement of science and the historical-philosophical analysis?

Logic is the study of the structure and principles of correct reasoning, and attempts to establish the principles that guarantee the validity of deductive arguments. The concept of validity is central to the logic, because, when we affirm the validity of an argument, we are saying it is impossible that its conclusion is false if its premises are true. This leads to two questions still controversial. So, what are the "bearers of truth"? The answers are multiple sentences, judgments, statements, propositions. And what is the truth? An issue far more radical and there it is no easy answer. Intuitively, we tend to assign to the truth two fundamental properties: universality and objectivity. Universality, as it is a common capacity of all humans, and objectivity because it is independent of personal considerations. We can all tell the truth, regardless of who, or what, we are. But both properties alone do not guarantee anything, not even differentiate the mere opinion or belief of the truth. But we cannot address an argument on the concept of truth, if we do not decide first what things can be true or false. We say that propositions are those who can be true or false. Propositions are descriptions of the world, are statements or denials of events in possible worlds. There is a long philosophical tradition of distinguishing between truth "necessary/a priori/logical" truths and "contingent/post/ factual". Both have led the two concepts of logical truth to not being opposed, but different from each other: the conception of truth as coherence, and the concept of truth as correspondence. From the point of view of consistency, a proposition is true or false as it relates to a given set of propositions, because it has consistently applied the rules of that system. Under the terms of correspondence, a proposition is true or false if it agrees with the reality referred to.

Other views have tried to go beyond this dichotomy, as the semantic point of view of Alfred Tarski (who lately does not stop being a variant of the matching concept), or the point of view of

the redundancy of Ramsey (which seeks to circumvent the problem considering the concept of truth as superfluous). To increase the complexity of the problem, we must declare not only the truth or falsity of propositions, but also that of theories, ideas and models. And therefore there are new conceptions of truth. The pragmatic conception (James) affirms the truth of those ideas that help us reach satisfactory relationships with other parts of our experience. The concept of falsifiability (Popper) tells us that a theory will be true if it is satisfactory to describe a given domain of reality. The basic idea underlying all these views (except perhaps in Karl Popper) is the intrinsic dichotomy between true and false. Recall that this opposition implies the validity of two fundamental laws of classical logic,

- Principle of excluded middle, i.e. a proposition is true or false and there is no other possibility.
- Principle of contradiction, i.e. no proposition is true and false simultaneously.

This basic idea and its two corollaries generate a series of paradoxes that are based on the need to overcome this strict truth-bivalence of classical logic. The future contingencies were dealt with by Aristotle, as interpreted by Jan Lukasiewicz, who pushes us towards fatalism. Accepting that a proposition about a future event is true or false (respectively) makes the event expressed by the proposition become necessary or impossible. The solution proposed by him in his classic work [Lukasiewicz, 1920] is the acceptance of a logic with three truth values, which in addition to true and false, accepts an indeterminate truth value. Naturally, the laws of excluded and not stop working contradiction, with the effects resulting therefrom, will be seen later.

The Principle of Uncertainty is a well-known Heisenberg's principle which appears in Quantum Mechanics. According to this, it is not possible to determine the position and momentum of a particle simultaneously. Heisenberg concludes from this that propositions are meaningless. Bart Kosko has much further and virulently criticized the interpretation of the uncertainty principle. Under this interpretation there is not enough trivalent logic. Kosko began to assimilate it to a driver driving on the motorway: the more you look, the best you know the road position, and conversely, the more you look, the better you know your speed odometer. But we cannot exclude any of the two skills.

9. More about the Origins of Non-Classical Logics

The paradoxes of quantity are known from the Presocratics (recall the Sorites Paradox, on the millet grain, of Zeno of Elea). Kosko defined them as the path from A to not A. Take away a grain of sand from a lot of it, and we still have a lot of sand. The problem is how many grains of sand must be removed, so that the pile ceases to be? The problem is not trivial, and involves many common expressions of predicates, as when we speak of "tall people". If we define "tall person" by measuring over six feet tall, are we saying that those who measure 1799 mm are short? Russell himself had already realized this when, in 1923, he published his paper, "Vagueness", in which questions about how many hairs does a person need to lose to be called bald. In this case, as in the previous, an answer is not sufficient, as in the first trivalent.

Many of the assertions we make as we consider that no more truths come from inductions as such, are never complete, for it can never be sure that we make the following observations contradict the assertion. "All men are mortal" is a statement provisionally true (there are still many millions of individuals who have not been verified). In these cases it may be more reasonable to speak of plausibility than truth. Almost all of the commonly accepted theories in the social sciences are plausible, acceptable, and in no case can be described as true. We accept them because they simplify our understanding of the world by reducing it to a manageable scheme that allows us to make future projections. Modern Chaos Theory attempts to introduce this "arrow of time" in the natural sciences, a factor which had always dispensed looking deterministic laws of nature.

From the above, it follows that you may need a radical rethink of the classical concepts of truth and falsehood, replacing the concept of vagueness, in which the truth and/or falsity are only extreme cases. By fuzzing understand the fact that a proposition can be both partially true and

partially false. A person is not simply tall or short, but part of both characteristics partially, so that only above and below will classify certain heights of the necessarily high or low, while in the intermediate zone of the two heights will exist a graduation which it ceases to be high. Intuitively, the concept of fuzziness is rooted in most of our modes of thinking and speaking. The introduction of such concept of fuzziness does not solve the problem of indeterminate sentences (future contingencies, for example), that is, devoid of truth value. The fuzzy principle states that everything is a matter of degree. All propositions acquire a truth value between one (true) and zero (false), inclusive. These extreme values are only given in the case of logical truths or untruths or strong inductions: "All men are mortal" can be an example of strong induction, since there is no counterexample.

The arguments for introducing the concept of fuzziness in logic have already been exposed, but it will be necessary to examine in detail some essential aspects before admission: the historical background and methodological concept; the possibility of constructing a formal language infinite-valued and, if necessary, define their properties and laws, and the philosophical and practical consequences of such an introduction.

Kosko highlights the differences between Eastern and Western philosophies regarding the concept of truth, summing it up in opposition, Aristotle Vs Buddha. Actually Kosko claims that Western philosophy, Aristotle's successor, has accepted uncritical as bivalence of simplifying system a reality too complex, what it has gained in simplicity lost in accuracy. In contrast, Eastern philosophies (Buddha, Lao Tsé, Confucius,…) have accepted since ancient times the strict unity of opposites, the well-known Yin and Yang.

Kosko's thesis may be difficult to verify because they lack many Eastern texts, unlike the case with Aristotle, of which we have the six works composing the Organon. However there are some indirect indications that allow us to believe in it, such as, for example, the list of the 320 BC Chinese paradoxes of Needham, some of which are very similar to those of Zeno of Elea. Harder Taoism seems to link with the concept of fuzziness rather because it is a metaphysical construct (which has curious similarities to the Plotinian Neoplatonism), in which the Tao is the guiding principle (the One of Plotinus) accessed by a mystical asceticism. On the other hand, if it is true that Aristotle was the great introducer of bivalence absolute, we must not overlook that he did not overlook aspects that can be fuzzy propositions as when he says: "In any case, what is said according to these (qualities) supports unquestionably the most and least", or as he does in the passage which defines abduction, or when he asserts that we can come to knowledge, but without the certainty of it. If Aristotle did not study this concept, it may be because he did not have the mathematical apparatus for developing a fuzzy logic. The gestation of this algebraic device starts with Newton's and Leibniz's developed calculus in the seventeenth century. The Aristotelian account of motion is replaced by the Newtonian due to calculus, without which it would not be possible. Today, Mathematical Analysis permeates all humans and natural sciences. This calculus introduces the question of degree: to what degree is it modified by the seamlessly changing of B, where A is a dependent variable of B?

The mathematization of logic, which begins in mid-nineteenth Augustus De Morgan and George Boole, has its turning point in the initiation of set theory in the late (Georg Cantor and Gotllob Frege), but almost immediately appear the first theory paradoxes (Bertrand Russell paradox of classes). In short, the concept of inclusion is nothing but a form of ambivalence: the elements are included or not included in the set, no middle ground. It was not until 1937 that Max Black wrote a first paper on vague sets, which had little or no influence. The final formalization of fuzzy sets is due to Lotfi Zadeh (1965), who creates an axiomatic.

It is debatable whether there is evidence of three-valued logic in Aristotle (future contingent), or William of Ockham (distinct knowledge/awareness confusing), but the fact is that the first three-valued logic was developed in 1909 by Vasiliev, eliminating the principle of the excluded middle Aristotelian logic. However, it is currently recognized as the first creator of three-valued logic Jan Lukasiewicz, who in 1920 proposed three truth values for propositions: true, false and indeterminate. Jan Lukasiewicz and Alfred Tarski himself infinite-valued logic developed later.

But fuzzy logic has not been fully axiomatized until it has had the concept of fuzzy set by Zadeh in 1965. The idea is simple: for any element of a set, its membership condition is given by a function m (x). When m (x) takes only two values, 1 (part) and 0 (not belong), then we have an ordinary set; when m (x) can take any discrete value within the range [0, 1], then we have a fuzzy set.

Let U be the set, possibly infinite, of all propositions. Let p, q, r, s, ..., elements of U (i.e. atomic propositions). In classical logic presuppose a mapping v of the set U in the set {0, 1}, such that v (p) = 0 when p is false, and v (p) = 1 when p is true. Because we establish a classification of propositions by the equivalence relation as follows, p R q, if v (p) = v (q). Under these circumstances and the relationships defined, we can ensure the set-theoretic structure of Boolean Algebra for all associated sets such that C (p) = {x ∈ U: x verifies p}. This allows us to reduce all the usual formulas of classical logic equations of the theory of sets.

Fuzzy sets and vague predicates: P is the set of predicates {P, Q, R, ...}. And U will be the set of objects in the universe of discourse, {x, y, z, ...}. The Cartesian product P x U represents the set of all dyadic propositions {Px, Py, Pz, ...; Qx, Qy, Qz, ...; Rx, Ry, Rz, ...}. We can only assign truth values in the set product, since both predicates and objects lack truth values. Using the semantic criterion of truth, we can establish the kind of real predicates (V) and false (F) for any predicate. But it happens that some propositions are not clearly true or false, but have a certain level. We define more classes than the classical F and V; so, we define the degree of compatibility of B with x. This will set the fuzzy subset P using the following expression: $x \in_a P$ if and only if mp (x) = a, where the symbol $\in_a$ to mean 'belongs in grade a', and the role mp represents the truth value of proposition Px for all x ∈U, and whose range is [0, 1]. But how is the membership function, mp? It is the fundamental problem: the criteria to be followed in order to establish the degree of allocation.

Somewhat problematic is also the definition of operators in the set P. L. Zadeh, which defined the operation conjunction "A and B" by (A and B) (x) = Max [A (x), B (x)], also symbolically “A or B” by the formula (A or B) (x) = Min [A (x), B (x)], that meets the associative, commutative, idempotent, increasing, continuous with 1 as unit and 0 as absorbing.

Denial is defined by an involutive decreasing bijection such that N: [0, 1] → [1, 0]. And from this the rule of disjunction can be obtained.

All constructions take us away from the structure of Boolean algebra, which, as we have ensured the conversion of logical rules, set algebra operations. Therefore, it will be desirable to establish criteria for measuring the fuzziness (fuzzy entropy) and conversion systems of fuzzy subsets in their classic closer subsets.

10. Logics and Computing

The computer systems are still in their basic structure, binary. There seems to be a contradiction between the principle fuzzy intuitively correct, and information processing systems and rules that are based on the 0 and 1.

B. Kosko opposes the concept of artificial intelligence (expert systems-building learn about it) to the neural network (creation of computer systems that simulate the human brain functions), a concept that has also been questioned from neurobiology.

It is true that the idea of truth/falsehood mutually exclusive is a simplification of the world, but the fact remains that any process of abstraction is in itself simplistic. Why not use the simplest, i.e. the bivalent? The scientific paradigm shifts only do the above theory denying and affirming the new.

Ilya Prigogine describes the latest contributions, and continues to pursue the search of deterministic laws for the explanation of the unstable and chaotic phenomena, introducing theories that Koko harshly attacks: "Once again, the scientists were wrong at 'self-evident' of logic and mathematics had inflated their hunches, instincts and conditioned reflexes of 'probability' and 'randomness' to make them a pagan god, and they had filled every corner of the universe as in the last century had filled all the invisible 'light ether' so that light waves could move through space".

Since the adoption of formal ethics perspective, from Kant, no one has come up with a system of moral foundation of the predicate. The foundation attempts through modern dialogic ethics (Apel, Habermas) to just collide with the fuzzy: moral statements are "fuzzified" to the core, because they may be obtained through surveys or through dialogue, mediated by our emotions and our interests . The introduction of the concept of fuzziness in this area not only increases the ethical relativism. But the construction of fuzzy sets lies in the formulation of the membership function, which is virtually impossible to deprive of subjectivism, in the same way that a judge cannot completely ignore his own ethical conceptions when applying justice, although it should be the ideal.

11. Conclusions

The knowledge and detailed analysis of these Non-Classic logics turns out to be undoubtedly essential for the development of new methods that allow to solve the sometimes extremely complex and difficult problems that Artificial Intelligence raises every day. With this article we have tried to provide an as wide as possible vision of those new tools of philosophical-logical-mathematical nature.

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