

## An Efficient Expert System Generator for Qualitative Feed-Back Loop Analysis

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### Abstract

Quite often the variables used in system analysis are qualitative in nature. They cannot be defined precisely, whereas software development for system analysis needs a mathematical framework with precise computations. It is not trivial to capture the uncertainty in the system. Fuzzy sets provide us the facility to capture the uncertainty in the system. In normal crisp set where the membership of an element is always certain in a sense that it would be member or not of the given set. In contrast to this a membership functions or possibility (ranging from 0 to 1, including both values) is assigned with each member. System analysis is done through system dynamics which is not very efficient. We present an efficient technique to generate expert system using fuzzy set. In our proposed approach five linguistic qualifiers are used for each variable, namely, Very Low (VL), Low (L), Medium (M), High (H), and Very High (VH). We capture the influence or feedback in the system with the help of if then else rules and matrices are generated for them which are used for analysis. Complete methodology and its applicability are presented here.

**Keywords:** Analysis, Applications, Design and Data Analysis, Fuzzy Mathematical Programming, Fuzzy System Models, Fuzzy Relations, Linguistic Modeling.

### 1. Introduction

Quite often the variables used in system analysis are qualitative in nature. They cannot be defined precisely, whereas software development for system analysis needs a mathematical framework with precise computations. It is not trivial to capture the uncertainty in the system. Fuzzy sets provide us the facility to capture the uncertainty in the system. In normal crisp set where the membership of an element is always certain in a sense that it would be member or not of the given set. In contrast to this a membership function or possibility (ranging from 0 to 1, including both values) is assigned with each member. System analysis is done through system dynamics which is not very efficient. We present an efficient technique to generate expert system using fuzzy set.

#### 1.1. Introduction to Fuzzy Sets

Since its inception in 1965 by Zadeh [1], fuzzy sets have seen an increasing exponential growth. It is a body of concepts and techniques that gave a form of mathematical precision to human cognitive process that in many ways are imprecise and ambiguous, by the standards of classical mathematics. These concepts are gaining a growing acceptability among engineers, scientists, mathematicians, linguists and philosophers.

An underlying philosophy of the theory of fuzzy sets is to provide a strict mathematical framework, where imprecise conceptual phenomenon in modelling and decision making may be precisely and rigorously studied. It provides for a gradual transaction from the realm of rigorous, quantitative and precise conceptions. It can also be considered as a modelling language and phenomena exist. According to Kaufman [2], fuzziness is s pervasive in human attempts to conceptualize, categorize, classify and relate all perceived phenomena, and thus arises the need of fuzzy set theory.

## **1.2. Need of Fuzzy Sets in System Analysis**

Managerial decision making involves a lot of uncertainties, that could be attributed to various reasons like beliefs and perceptions of managers, uncertain futuristic situations etc. Quite often the knowledge of managers is imprecise and vague. Similarly, the managerial systems are also comprised of a lot of subjective variables, which are not easily quantifiable with the classical mathematical treatment based on crisp sets. Thus much of the information in the knowledge base for such system is imprecise, incomplete or not totally reliable, and management of uncertainty becomes a critical issue in the design of knowledge based systems. In the existing expert systems, uncertainty is dealt with through a combination of predicate logic and probability based methods. Such methods are not perfect and have a serious shortcoming that they are not capable of coming to grip with the pervasive fuzziness of information in the knowledge base and consequently are mostly adhoc.

A very critical requirement of expert support systems is that of effectively and deeply embedded user interfaces that would enable their human users to easily inspect and control the problem solving process. Since managers use natural language most of the time, such user's interfaces should be able to understand and interpret the natural language concepts. In case of natural language, the extension of a predicate is in general a fuzzy set in which the transition from membership to non membership is gradual rather than abrupt. Almost all the concepts in or about natural languages are fuzzy in nature [3]. Thus fuzzy set theoretical approach could be made use of, thereby providing a valuable contribution to user interfaces that enable the user to converse with the computer in a natural language like English.

## **1.3. Problem Definition**

### **1.3.1. Objective of the Study**

The main objective of the study is to develop an interactive and user friendly software package for qualitative analysis of systems using fuzzy sets. The second objective of the study is to do further methodological developments in the existing methodology for causal loop generation and qualitative analysis of systems using fuzzy sets.

### **1.3.2. Issues Covered in this Study**

- Creation of feedback loops depending on the causal relationships.
- Define all the linguistic variables.
- Creation of *if.... then* rules for causal relationships.
- Creation of fuzzy matrices for above relationships.
- Qualitative analysis of the causal loop structure in the forward direction using fuzzy sets theoretical concepts.
- Converting possibility distribution into proper linguistic output.

### **1.3.3. Scope of this Study**

The scope of this study is limited to develop an interactive and user friendly software package for qualitative analysis of systems in forward direction only, using fuzzy sets. The output may give guidelines for designing a package for backward analysis.

## **1.4. Organization of this Paper**

Related work reported in literature is reported in the next Section. Section 3 describes the proposed methodology. A case study is presented in the Section 4. The paper is concluded with conclusions and directions for future work in the last Section.

## 2. Related Work

### 2.1. Introduction to System Dynamics

The system dynamics (SD) approach with its causal philosophy and purpose of gaining deep insight into the working of a system places heavy emphasis on “looking within the system”. There are mainly six problem solving steps through which the system dynamics methodology operates, such as: Problem identification and definition, system conceptualization, model formulation, simulation and validation of the model, policy analysis and improvement, policy implementation. In relation to SD researches, many applications in different fields which have diversity can be seen. For example system thinking and modeling for a complex world [4], applying system dynamics with data mining in a biological model [5], Navigating towards sustainable development: A system dynamics approach [6], Knowledge based decision making on higher level strategic concerns: system dynamics approach [7] and etc. Also the applications and implementations of fuzzy logic in different sciences and industrial problems are much because of its popularity to deals with vague and imprecise information [8], [9]. However the number of studies is focused on application of fuzzy logic in the system dynamics is not large amount. Pervious researches for this approach, which are considerable, are Tahera.k & Ibrahim.R.N [9], Seçkin polat & Cafer Erhan [10], Vassilios Karavezyris & Klaus Peter [11].

SD as developed by Forrester [12] is a problem solving approach to complex problems which emphasize the structural aspects of model of systems. It can be used to modeling and simulation real world systems. SD deals with the time dependent behavior of managed systems with the aim of describing the system and understanding, through qualitative and quantitative models, how information feedback governs its behavior, and designing robust information feedback structures and control policies simulation and optimization [13]. SD consists of some components: Causal loop diagram, Stock and Flow diagram and Level and Rate variables.

### 2.2. Fuzzy Logic Approach

Fuzzy sets were introduced by Zadeh [1] as an extension of the classical set theory. The fuzzy logic was built around the central concept of a fuzzy sets or membership functions. Fuzzy logic expresses vague and subjective relationship mathematically. It is a suitable way to mapping an input space to an output space using a fuzz set. The conventional set theory classifies the elements of the set into crisp set. The membership degree is One, when an element belongs to the set and zero, when an element does not belong to the set. Unlike the classical set theory, in the fuzzy set theory, an element can belong to a fuzzy set with the membership degree between zero and one. Figure 1 shows the difference between a classical set and a fuzzy set. In the classical set, 30C is considered as cold whereas in the fuzzy set, the degree of coldness for 30C is 0.85 and the degree of hotness for 30C is 0.15, i.e. 30C is cold by 85% and hot by 15%.

A membership function is a curve that represents the degree of points which belong to the specific fuzzy variable. Selecting the appropriate membership function plays an essential rule in design of a fuzzy logic controller. The shape of membership function could be defined based on the simplicity, convenience, speed and efficiency. Many different membership functions are introduced in the literatures such as triangular, trapezoidal and Gaussian. The membership function which represented in figure 1(b) is a trapezoidal type.

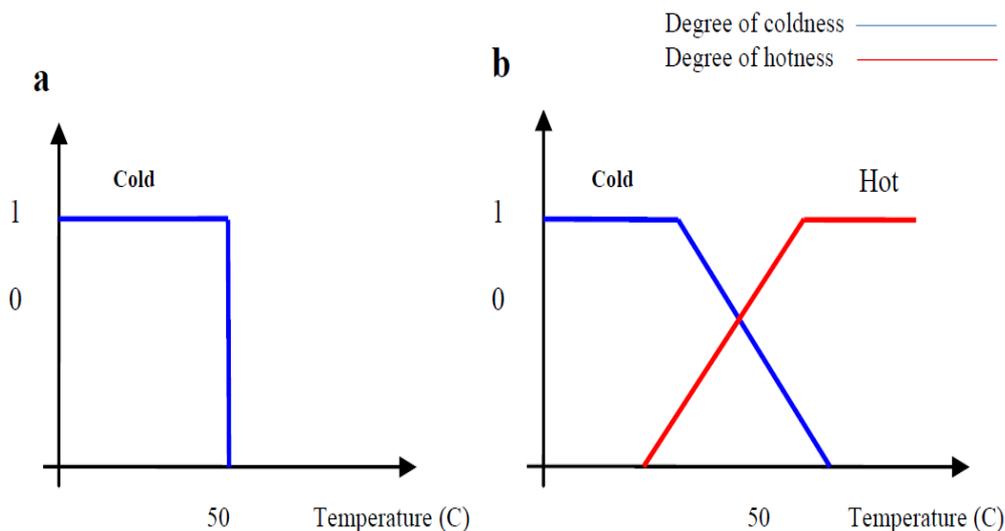


Figure 1. (a) Classical set and (b) Fuzzy set

### 2.3. Fuzzy Rule Based

Fuzzy rule based is developed to relate the input variables to the output variables by if-then rules. Fuzzy rules consist of two parts: an antecedent part stating condition on the input variables, and a consequent part describing the related value of the output variables. A single fuzzy if-then rule assumes as follow:

if  $x$  is A then  $y$  is B

Where A and B, are linguistic values defined by fuzzy set on the ranges (universes of discourse) X and Y, respectively. The if- part of the rule "  $x$  is A " is antecedent or premise, while the then part of the rule "  $y$  is B " is consequent or conclusion. An example such a rule might be If service is good then price of food is average In this rule, service and price are two fuzzy variables. The fuzzy values in this rule are good and average descriptions. These values are defined on universes of discourse and determine the degree of element  $x$  which belongs to the membership functions. In general, the input to an if-then rule is the current value for the input variable (in this case, service) and the output is an entire fuzzy set (in this case, average). This set will later be defuzzified i.e. assigning one value to the output. The concept of defuzzification is described in the next section. In some case the antecedent or consequent of a rule can also have multiple parts. Therefore given particular values of the input variables, the degree of fulfillment of a rule is obtained by aggregating the membership degrees of these input values into the respective fuzzy sets. The fuzzy output is determined by the degrees of fulfillment and the consequent parts of the rules. The most common ways to modify the output fuzzy set are truncation using the *min* (minimum) function or scaling using the *prod* (product) function.

### 2.4. Fuzzy Inference Systems

Fuzzy inference is the process of formulating the mapping from given inputs to an output using fuzzy logic, the mapping then provides a basis from which a decision can be made. It used for evaluating fuzzy linguistic descriptions using concepts such as membership functions, fuzzy logic operators, and if-then rules [14]. Because of its multidisciplinary nature, FIS are associated with a number of names, such as fuzzy expert systems, fuzzy modeling, fuzzy associative memory, fuzzy logic controllers, and simply fuzzy systems. In general there are two common types of FIS: *mamdani* and *sugeno* types [8]. These two types of inference systems vary somewhat in the way outputs are determined. For these two types of FIS, there are entire descriptions in the literatures. [15], [16], [17].

## **2.5. Application of fuzzy logic for modelling System Dynamics**

If SD has at least one of the below conditions, can be modelled with fuzzy logic.

- Some of the level or rate or other variables are fuzzy.
- The time factor has vagueness.
- Some of the relations among variables can be replaced by conditional statements, which contain fuzzy variables. These conditional statements usually are in the if-then format.
- The degree of uncertainty of variables can be represented by fuzzy probabilities, when the information on hand, is imprecise or incomplete.
- Some of the operators may be fuzzy.

According to these properties the fuzzy logic approach is used to modelling SD. We are working in preparing a framework for qualitative analysis of systems which may contain feedback loops. Since the qualitative inputs and outputs can be better handled using fuzzy sets, we have used fuzzy logic based approach in our work. We have developed complete approach and is being presented here. A complete case study is included to support our proposed approach.

## **3. Methodology**

### **3.1. Fuzzy Relation Model**

Not all the situations are very clear and transparent in managerial systems. Very often managers have to deal with uncertainties and vagueness. Such kinds of systems are not amenable to traditional approaches of crisp mathematical treatment and in such cases a fuzzy set theoretical approach appears to be the most appropriate choice. The fuzzy relation model provides an alternative method for qualitative analysis of causal loop structure. It incorporates the uncertainties that could be attributed to the beliefs and perceptions of the user, and models them using various fuzzy set theoretical concepts like linguistic variables, modus ponens's rule, fuzzy relation composition, possibility distribution etc.

Three different kinds of qualitative analysis are possible through such an approach. The first is that of forward analysis to see the effects of changes in some variables on those variables which are affected by them using the maximum composition over the relations falling on the particular loop consisting of these variables. The answers are provided in terms of linguistic variables which correspond to the common language of managers are the highly appealing. The second kind of analysis makes use of the possibility theory. Possibility distributions are assigned to causal links, which on further analysis helps in estimating the possibility of achieving a desired state of existence. The third kind of approach makes use of the fuzzy relation equation, for the backward analysis of the causal loop structure. It shows how the values of various parameters and variables, to reach the desired state of existence, could be determined using fuzzy relations.

Besides all these advantages, a great benefit of the fuzzy relation model is that, it could be used for deriving an expert system structure for system dynamics model. It also provides deep insight into the structure and behaviour of the system and feed back obtained from it could be incorporated into causal loop structure.

### **3.2. Defining Linguistic Variables**

For every variable all the linguistic variables (in our case, namely, very low, low, medium, high and very high) are to be defined by assigning different possibility distribution for a set of reference data values for variables (we had taken 4).

#### **3.2.1. Development of causal loop structure**

The key variables which explain the way the system works are identified from the verbal description. These loops are drawn using arrows connecting the variables. The direction of arrow denotes the direction of casualty. A set of variables lying on a circular path of casualty constitute a feedback loop. There are two types of feedback loops, namely, positive and negative. To identify the polarity of loops, i.e. whether the loop is positive or negative, pair wise relationships of relevant variables are established. Going round the loop, when a change in one variable produces change in

the same direction in another variable, the relationship is defined as positive. When the change in another variable runs in the opposite direction, the relationship is defined as negative.

A reinforcement of change between two variables is indicated by placing a positive sign on the arrow head, and opposite to change is denoted by a negative sign. The polarity of a loop is determined by tracing the loop, assuming that everything else remain constant. Going around the loop, if all signs are positive, or negative signs are even in number, the loop polarity is positive, otherwise it is negative. The polarity of the loop is shown in the loop diagram, by making a positive or negative sign in the centre of the loop, surrounded by a circular drawn arrow showing the direction is called the feedback loop structure.

In a positive feedback loop, a variable continually feeds back upon it self to reinforce the process of its growth or collapse. This loop can only act to accelerate the initial change introduced in any variable. Without constraints introduced by interaction with other loops, each positive feedback loop produces exponential change of all variables lying on it.

A negative feedback loop acts to counter the direction of initial change in any of its variables. It is characterized by goal directed or good seeking behavior. As compared to positive feedback loops, negative feedback loops have a variety of behaviors. These loops tend to keep a given system at equilibrium. The simplest form of negative feedback loop behavior is gradual adjustment, without oscillation or fluctuation, i.e. asymptotic growth or asymptotic decay towards a goal. Damped oscillation is another kind of behaviour as exhibited by these loops.

### 3.2.2. Compositional rule of inference

Let  $X$  and  $Y$  be two universes of discourses with base variables  $x$  and  $y$  respectively. Let  $R(x)$ ,  $R(x,y)$  and  $R(y)$  denote restriction on  $x$  and  $y$  respectively, with the understanding that  $R(x)$ ,  $R(x,y)$  and  $R(y)$  are fuzzy relations in  $X$ ,  $X \times Y$  and  $Y$ . Let  $A$  and  $F$  denote particular fuzzy subsets of  $X$  and  $X \times Y$ . Then the compositional rule of inference asserts that the solution of the relational assignment equations (Zadeh 1976).

$R(x) = A$  and  $R(x,y) = F$  is given by  $R(y) = A * F$  where  $A * F$  is the max-min composition of  $A$  and  $F$ , as explained as follows. In this sense we can infer  $R(y) = A * F$  from  $R(x) = A$  and  $R(x,y) = F$ .

Let  $R(x,y)$ ,  $(x,y) \in X \times Y$  and  $S(y,z) \in Y \times Z$  be two fuzzy relations. Then the max-min composition of  $R$  max-min  $S$  denoted by  $R * S$  is

$$R * S = \{[(x,z) \max \text{ over } y \{ \min \{ \mu_R(x,y), \mu_S(y,z) \} \}] \mid x \in X, y \in Y, z \in Z\}$$

Here  $\mu$  represents possibility of membership of a set.

### 3.2.3. Feedback Loop Analysis

Impact of change of value of any variable on the other variables in the system is evaluated on the basis of relationships between the variables which we have now represented in form of fuzzy matrices. Because of feedback loop the starting variable itself will be modified so the process would be repeated to know the impact of this new value of that particular variable. The process is iterated till the system is stabilized.

### 3.2.4. Conversion of output into Linguistic form

For interpretation of results in linguistic (user will be interested in this type of output only) form, deterministic approach is used and the interpolation method is used for giving specific outputs. This is done for all the variables. For each variable five linguistic qualifiers, namely, very low, low, medium, high and very high are defined by assigning different possibility distributions four data values of that variable. For each definition, the virtual value (Note that deterministic approach is used here! Fuzzy logic is needed here also!!) is calculated by the formula

$$E.V. = (p_1x_1 + p_2x_2 + p_3x_3 + p_4x_4) / (p_1 + p_2 + p_3 + p_4)$$

Where  $p_i$ 's and  $x_i$ 's are possibility and data value respectively for  $i=1, 2, 3$  and  $4$ .

Now we got five levels of values. There are four gaps, between very low and low, between low and medium, between medium and high and between high and very high. We have dividing each gap into six equal parts. Then we defined thirteen different linguistic qualifiers, one for each part. These are very low, slightly higher than very low, somewhat low, low, slightly higher than low, somewhat medium, medium, slightly higher than medium, somewhat high, high, slightly higher than high, somewhat very high and very high.

At the end of analysis, when we get equilibrium, each variable will have four possibilities  $p_1$ ,  $p_2$ ,  $p_3$  and  $p_4$  for four data values  $x_1$ ,  $x_2$ ,  $x_3$  and  $x_4$  respectively. Now E.V. (estimated value) is calculated as per the formula described earlier. Based on the slot in which this value is lying, proper linguistic output is given.

#### 4. Case Study

##### 4.1. Problem Situation

It has been observed that the demand is affected by the market size in a positive manner. The market size in turn is affected by disposable income per capita. Any increase in demand decreases the average fixed cost fraction, thereby reducing the price. The price affects the market size in a negative manner. Also, the sales revenue is affected by the demand in a positive manner and an increase in the sales revenue leads to an increase in the advertising budget which in turn reinforces the demand.

##### 4.2. Causal Loop Diagram

The problem situation could be represented in the following form of a causal loop diagram as shown in Figure 2.

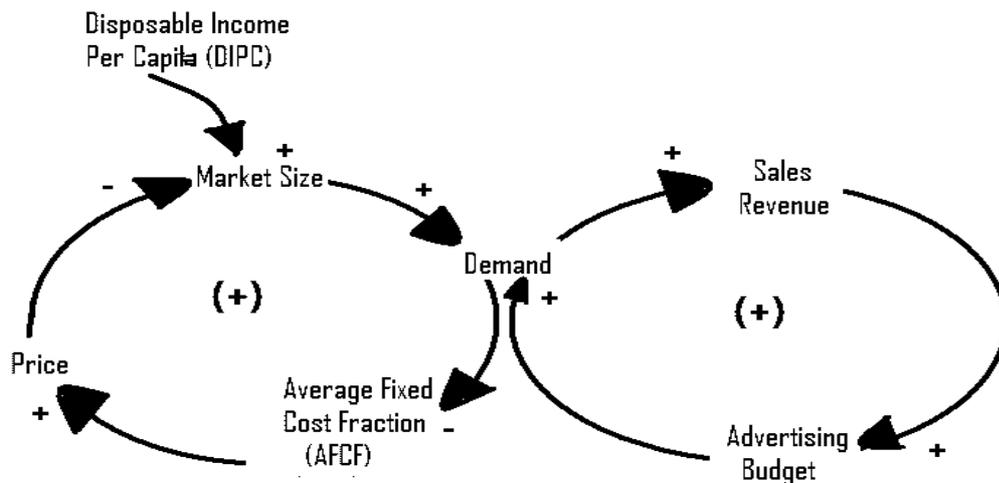


Figure 2. Causal loop diagram for the problem situation

##### 4.3. Data

The range of various variables as specified by the manager is shown in table 1.

Table 1. Range of Variables

DIPC	MARKET SIZE	PRICE	AFCF	DEMAND	SALES REVENUE	ADVERT. BUDGET
(Rs./Person/Week)	(Units/Week)	(Rs./Unit)	(Rs./Unit)	(Units/Week)	(Rs./Week)	(Rs./Week)
500	3500	4.8	0.050	400	2000	50
550	4500	4.9	0.125	700	3500	600
600	5500	5.0	0.200	1000	5000	1150
650	6500	5.1	0.275	1300	6500	1700

#### 4.4. Definitions of Linguistic Variables

We have defined five linguistic variables for each variable, namely, Very Low (VL), Low (L), Medium (M), High (H) and Very High (VH). Definitions for the variables used in our case study are presented.

<i>DISPOSABLE INCOME PER CAPITA (DIPC)</i>				
VL	= 0.8/500	+ 0.4/550	+ 0.0/600	+ 0.0/650
L	= 1.0/500	+ 0.8/550	+ 0.0/600	+ 0.0/650
M	= 0.0/500	+ 1.0/550	+ 0.4/600	+ 0.0/650
H	= 0.0/500	+ 0.6/550	+ 1.0/600	+ 0.6/650
VH	= 0.0/500	+ 0.0/550	+ 0.6/600	+ 1.0/650

<i>MARKET SIZE</i>				
VL	= 0.8/3500	+ 0.5/4500	+ 0.0/5500	+ 0.0/6500
L	= 1.0/3500	+ 0.8/4500	+ 0.5/5500	+ 0.0/6500
M	= 0.8/3500	+ 1.0/4500	+ 0.5/5500	+ 0.0/6500
H	= 0.0/3500	+ 0.3/4500	+ 0.6/5500	+ 1.0/6500
VH	= 0.0/3500	+ 0.0/4500	+ 0.4/5500	+ 0.7/6500

<i>PRICE</i>				
VL	= 0.8/4.8	+ 0.6/4.9	+ 0.0/5.0	+ 0.0/5.1
L	= 0.8/4.8	+ 1.0/4.9	+ 0.5/5.0	+ 0.0/5.1
M	= 0.0/4.8	+ 0.8/4.9	+ 1.0/5.0	+ 0.8/5.1
H	= 0.0/4.8	+ 0.5/4.9	+ 0.8/5.0	+ 1.0/5.1
VH	= 0.0/4.8	+ 0.0/4.9	+ 0.3/5.0	+ 0.7/5.1

<i>AVERAGE FIXED COST FRACTION (AFCF)</i>				
VL	= 0.7/0.050	+ 0.6/0.125	+ 0.0/0.200	+ 0.0/0.275
L	= 0.8/0.050	+ 1.0/0.125	+ 0.7/0.200	+ 0.0/0.275
M	= 0.0/0.050	+ 0.7/0.125	+ 1.0/0.200	+ 0.7/0.275
H	= 0.0/0.050	+ 0.6/0.125	+ 0.8/0.200	+ 1.0/0.275
VH	= 0.0/0.050	+ 0.0/0.125	+ 0.4/0.200	+ 0.7/0.275

<i>DEMAND</i>				
VL	= 0.8/400	+ 0.6/700	+ 0.0/1000	+ 0.0/1300
L	= 1.0/400	+ 0.8/700	+ 0.5/1000	+ 0.0/1300
M	= 0.0/400	+ 1.0/700	+ 0.6/1000	+ 0.0/1300
H	= 0.0/400	+ 0.7/700	+ 1.0/1000	+ 0.6/1300
VH	= 0.0/400	+ 0.0/700	+ 0.6/1000	+ 0.9/1300

<i>SALES REVENUE</i>				
VL	= 0.8/2000	+ 0.3/3500	+ 0.0/5000	+ 0.0/6500
L	= 1.0/2000	+ 0.5/3500	+ 0.0/5000	+ 0.0/6500
M	= 0.0/2000	+ 1.0/3500	+ 0.7/5000	+ 0.0/6500
H	= 0.0/2000	+ 0.8/3500	+ 1.0/5000	+ 0.0/6500
VH	= 0.0/2000	+ 0.0/3500	+ 0.6/5000	+ 0.9/6500

<i>ADVERTISING BUDGET</i>				
VL	= 0.7/50	+ 0.5/600	+ 0.0/1150	+ 0.0/1700
L	= 1.0/2000	+ 0.8/3500	+ 0.0/5000	+ 0.0/6500
M	= 0.8/2000	+ 1.0/3500	+ 0.0/5000	+ 0.0/6500
H	= 0.0/2000	+ 0.7/3500	+ 1.0/5000	+ 0.0/6500
VH	= 0.0/2000	+ 0.0/3500	+ 0.5/5000	+ 0.7/6500

Relation is captured in form of IF ... THEN..... ELSE IF..... ELSE form from the user. Using the fuzzy sets defined for linguistic qualifiers, fuzzy relation matrix is generated using max-min composition as described in the next subsection.

#### 4.5. Specification of Rules

The problem situation specifies only the positive or negative nature of the relationship. Some more detail information is captured in form of if ... then rules. The Manager has come out with the following set of rules that hold good for this particular case.

##### R1 (BETWEEN PRICE AND MARKET SIZE):

IF	PRICE is	Very Low	THEN	MKT SIZE is	Very High
ELSE IF	PRICE is	Low	THEN	MKT SIZE is	High
ELSE IF	PRICE is	Medium	THEN	MKT SIZE is	Medium
ELSE IF	PRICE is	High	THEN	MKT SIZE is	Low
ELSE IF	PRICE is	Very High	THEN	MKT SIZE is	Very Low

##### R2 (BETWEEN DIPC AND MARKET SIZE):

IF	DIPC is	Very Low	THEN	MKT SIZE is	Medium
ELSE IF	DIPC is	Low	THEN	MKT SIZE is	Medium
ELSE IF	DIPC is	Medium	THEN	MKT SIZE is	Medium
ELSE IF	DIPC is	High	THEN	MKT SIZE is	Medium
ELSE IF	DIPC is	Very High	THEN	MKT SIZE is	High

##### R3 (BETWEEN MARKET SIZE AND DEMAND):

IF	MKT SIZE is	Very Low	THEN	DEMAND is	Medium
ELSE IF	MKT SIZE is	Low	THEN	DEMAND is	Medium
ELSE IF	MKT SIZE is	Medium	THEN	DEMAND is	High
ELSE IF	MKT SIZE is	High	THEN	DEMAND is	Very High
ELSE IF	MKT SIZE is	Very High	THEN	DEMAND is	Very High

##### R4 (BETWEEN DEMAND AND SALES REVENUE):

IF	DEMAND is	Very Low	THEN	SALES REV is	Very Low
ELSE IF	DEMAND is	Low	THEN	SALES REV is	Low
ELSE IF	DEMAND is	Medium	THEN	SALES REV is	Medium
ELSE IF	DEMAND is	High	THEN	SALES REV is	High
ELSE IF	DEMAND is	Very High	THEN	SALES REV is	Very High

##### R5 (BETWEEN SALES REVENUE AND ADVERTISING BUDGET):

IF	SALES REV is	Very Low	THEN	ADV BGT is	Very Low
ELSE IF	SALES REV is	Low	THEN	ADV BGT is	Low
ELSE IF	SALES REV is	Medium	THEN	ADV BGT is	Medium
ELSE IF	SALES REV is	High	THEN	ADV BGT is	Medium
ELSE IF	SALES REV is	Very High	THEN	ADV BGT is	Very High

##### R6 (BETWEEN ADVERTISING BUDGET AND DEMAND):

IF	ADV BGT is	Very Low	THEN	DEMAND is	Medium
ELSE IF	ADV BGT is	Low	THEN	DEMAND is	Medium
ELSE IF	ADV BGT is	Medium	THEN	DEMAND is	Medium
ELSE IF	ADV BGT is	High	THEN	DEMAND is	Medium
ELSE IF	ADV BGT is	Very High	THEN	DEMAND is	High

##### R7 (BETWEEN DEMAND AND AFCF):

IF	DEMAND is	Very Low	THEN	AFCF is	Very High
ELSE IF	DEMAND is	Low	THEN	AFCF is	High
ELSE IF	DEMAND is	Medium	THEN	AFCF is	Medium
ELSE IF	DEMAND is	High	THEN	AFCF is	Low
ELSE IF	DEMAND is	Very High	THEN	AFCF is	Very Low

##### R8 (BETWEEN AFCF AND PRICE):

IF	AFCF is	Very Low	THEN	PRICE is	Very Low
ELSE IF	AFCF is	Low	THEN	PRICE is	Low
ELSE IF	AFCF is	Medium	THEN	PRICE is	Medium
ELSE IF	AFCF is	High	THEN	PRICE is	High
ELSE IF	AFCF is	Very High	THEN	PRICE is	Very High

#### 4.6. Generating Mathematical Framework for FUZZY Relation

Considering the linguistic definitions of different variables and relation in form of if ..... then rules, a suitable matrix is generated for each relation. Here matrix generation for relation R1 which is between PRICE and MARKET SIZE is presented here.

This relation R1 is converted into mathematical form in the following manner.

$$R1 = (0.8/4.8+0.6/4.9) * (0.4/5500+0.7/6500) + (0.8/4.8+1/4.9+0.5/5.0) * (0.3/4500+0.6/5500+1/6500) + (0.8/4.9+1/5.0+0.8/5.1) * (0.8/3500+1/4500+0.5/5500) + (0.5/4.9+0.8/5.0+1/5.1) * (1/3500+0.8/4500+0.2/5500) + (0.3/5.0+0.7/5.1) * (0.8/3500+0.5/4500)$$

$$R1 = 0.4/(4.8,5500) + 0.7/(4.8,6500) + 0.4/(4.9,5500) + 0.6/(4.9,6500) + 0.3/(4.8,4500) + 0.6/(4.8,5500) + 0.8/(4.8,6500) + 0.3/(4.9,4500) + 0.6/(4.9,5500) + 1.0/(4.9,6500) + 0.3/(5.0,4500) + 0.5/(5.0,5500) + 0.5/(5.0,6500) + 0.8/(4.9,3500) + 0.8/(4.9,4500) + 0.5/(4.9,5500) + 0.8/(5.0,3500) + 1.0/(5.0,4500) + 0.5/(5.0,5500) + 0.8/(5.1,3500) + 0.8/(5.1,4500) + 0.5/(5.1,5500) + 0.5/(4.9,3500) + 0.5/(4.9,4500) + 0.2/(4.9,5500) + 0.8/(5.0,3500) + 0.8/(5.0,4500) + 0.2/(5.0,5500) + 1.0/(5.1,3500) + 0.8/(5.1,4500) + 0.2/(5.1,5500) + 0.3/(5.0,3500) + 0.3/(5.0,4500) + 0.7/(5.1,3500) + 0.5/(5.1,4500)$$

$$R1 = 0.0/(4.8,3500) + 0.3/(4.8,4500) + 0.6/(4.8,5500) + 0.8/(4.8,6500) + 0.8/(4.9,3500) + 0.8/(4.9,4500) + 0.6/(4.9,5500) + 1.0/(4.9,6500) + 0.8/(5.0,3500) + 1.0/(5.0,4500) + 0.5/(5.0,5500) + 0.5/(5.0,6500) + 1.0/(5.1,3500) + 0.8/(5.1,4500) + 0.5/(5.1,5500) + 0.0/(5.1,6500)$$

Equivalently this relation **R1** can be represented in the matrix form as following.

*Matrix for R1 (BETWEEN PRICE AND MARKET SIZE)*

		MARKET SIZE			
		3500	4500	5500	6500
PRICE	4.8	0.0	0.3	0.6	0.8
	4.9	0.8	0.8	0.6	1.0
	5.0	0.8	1.0	0.5	0.5
	5.1	1.0	0.8	0.5	0.0

Similarly matrices are generated for all the causal relations present in the system. Such matrices for other relations are presented here.

*Matrix for R2 (BETWEEN DIPC AND MARKET SIZE)*

		MARKET SIZE			
		3500	4500	5500	6500
DIPC	500	0.8	1.0	0.5	0.0
	550	0.8	1.0	0.5	0.0
	600	0.8	1.0	0.6	0.6
	650	0.6	0.6	0.6	1.0

*Matrix for R3 (BETWEEN MARKET SIZE AND DEMAND)*

		DEMAND			
		400	700	1000	1300
MARKET SIZE	3500	0.0	1.0	0.8	0.6
	4500	0.0	0.8	1.0	0.6
	5500	0.0	0.5	0.6	0.6
	6500	0.0	0.0	0.6	0.9

Matrix for R4 (BETWEEN DEMAND AND SALES REVENUE)

		SALES REVENUE			
		2000	3500	5000	6500
DEMAND	400	1.0	0.5	0.0	0.0
	700	0.8	1.0	0.7	0.0
	1000	0.5	0.8	1.0	0.6
	1300	0.0	0.6	0.6	0.9

Matrix for R5 (BETWEEN SALES REVENUE AND ADVERTISING BUDGET)

		ADVERTISING BUDGET			
		50	600	1150	1700
SALES REVENUE	2000	1.0	0.8	0.0	0.0
	3500	0.8	1.0	0.8	0.0
	5000	0.7	0.7	1.0	0.6
	6500	0.0	0.0	0.5	0.7

Matrix for R6 (BETWEEN ADVERTISING BUDGET AND DEMAND)

		DEMAND			
		400	700	1000	1300
ADVERTISING BUDGET	50	0.0	1.0	0.6	0.0
	600	0.0	1.0	0.6	1.0
	1150	0.0	1.0	0.6	0.5
	1700	0.0	0.7	0.7	0.6

Matrix for R7 (BETWEEN DEMAND AND AFCF)

		AFCF			
		0.050	0.125	0.200	0.275
DEMAND	400	0.0	0.6	0.8	1.0
	700	0.7	0.7	1.0	0.8
	1000	0.8	1.0	0.7	0.6
	1300	0.7	0.6	0.6	0.0

Matrix for R8 (BETWEEN AFCF AND PRICE)

		PRICE			
		3500	4500	5500	6500
AFCF	0.050	0.8	0.8	0.5	0.0
	0.125	0.8	1.0	0.7	0.7
	0.200	0.7	0.8	1.0	0.8
	0.275	0.0	0.7	0.8	1.0

#### 4.7. System Analysis

Impact of change of value of any variable on the other variables in the system is evaluated on the basis of relationships between the variables which we have now represented in form of fuzzy matrices. Because of feedback loop the starting variable itself will be modified so the process would be repeated to know the impact of this new value of that particular variable. The process is iterated till the system is stabilized.

Now, Suppose manger wants to know the effect of Very Low PRICE in the system, the exogenous variable DIPC remains Low.

We know the definition of Very Low PRICE as following.

$$0.8/4.8 + 0.6/4.9 + 0.0/5.0 + 0.0/5.1$$

We obtain the effect of Very Low PRICE on the MARKET SIZE by using the corresponding matrix defined earlier, and by min max composition in the following manner.

PRICE	PRICE MARKET SIZE Relation Matrix			
<b>[0.8 0.6 0.0 0.0] *</b>	<b>0.0</b>	<b>0.3</b>	<b>0.6</b>	<b>0.8</b>
	<b>0.8</b>	<b>0.8</b>	<b>0.6</b>	<b>1.0</b>
	<b>0.8</b>	<b>1.0</b>	<b>0.5</b>	<b>0.5</b>
	<b>1.0</b>	<b>0.8</b>	<b>0.5</b>	<b>0.0</b>

This max min operation is performed in the following manner. When we are performing operation using PRICE row matrix and first column of relation matrix we get value V1 which will be first column value of the result matrix.

$$\begin{aligned} V1 &= \max(\min(0.8,0.0), \min(0.6, 0.8), \min(0.0, 0.8), \min(0.0, 1.0)) \\ &= \max(0.0, 0.6, 0.0, 0.0) \\ &= 0.6 \end{aligned}$$

Similarly V2, V3 and V4 are calculated using second, third and fourth column of relation matrix as following.

$$\begin{aligned} V2 &= \max(\min(0.8,0.3), \min(0.6, 0.8), \min(0.0, 1.0), \min(0.0, 0.8)) \\ &= \max(0.3, 0.6, 0.0, 0.0) \\ &= 0.6 \\ V3 &= \max(\min(0.8,0.6), \min(0.6, 0.6), \min(0.0, 0.5), \min(0.0, 0.5)) \\ &= \max(0.6, 0.6, 0.0, 0.0) \\ &= 0.6 \\ V4 &= \max(\min(0.8,0.8), \min(0.6, 1.0), \min(0.0, 0.5), \min(0.0, 0.0)) \\ &= \max(0.8, 0.6, 0.0, 0.0) \\ &= 0.8 \end{aligned}$$

This means the result matrix will be as follows.

$$[0.6 \quad 0.6 \quad 0.6 \quad 0.8]$$

It will be interpreted as follows.

$$0.6/3500+0.6/4500+0.6/5500 +0.8/6500$$

In this way, we have obtained the effect of Very Low PRICE on the MARKET SIZE and the result possibility distribution for market size obtained is [0.6 0.6 0.6 0.8]. But our analysis is not complete, because MARKET SIZE is also being affected by DIPC and DIPC is kept at LOW. We have following definition for Low DIPC.

$$\text{Low DIPC: } 1.0/500 + 0.8/550 + 0.0/600 + 0.0/650$$

This effect is calculated using another corresponding matrix and by doing that, the possibility distribution for market size obtained is [0.8 1.0 0.5 0.0].

Since the MARKET SIZE is affected by two variables PRICE and DIPC, its current final values of possibilities are found by getting intersection of possibilities (picking minimum).

$$[0.6 \ 0.6 \ 0.6 \ 0.8] \cap [0.8 \ 1.0 \ 0.5 \ 0.0] = [0.6 \ 0.6 \ 0.6 \ 0.0]$$

Hence the current possibilities at this stage will be [0.6 0.6 0.6 0]. Similarly possibilities of all the variables are calculated. In next iteration, the intersection of newly computed possibilities is done with its previous iteration's possibilities to get current possibilities at this stage.

If the exogenous variable (in this case DIPC) is constant, then process will complete when possibility for all variables stabilize. Iteration will be complete when the effect of all causal links has calculated in that particular iteration. If the exogenous variable is varying with time then iterations will be calculated till the qualitative input for the exogenous variable is given. In our example, the exogenous variable DIPC is constantly Low. Possibilities of all the variables stabilized in two iterations. The final possibilities distribution is as follows.

Variable Name	Value [Possibility]			
DIPC	500 [1.0],	550 [0.8],	600 [0.0],	650 [0.0]
MKT SIZE	3500 [0.6],	4500 [0.6],	5500 [0.5],	6500 [0.0]
PRICE	4.8 [0.6],	4.9 [0.6],	5.0 [0.0],	5.1 [0.0]
AFCF	0.050 [0.6],	0.125 [0.6],	0.200 [0.6],	0.275 [0.6]
DEMAND	400 [0.0],	700 [0.6],	1000 [0.6],	1300 [0.6]
SALES REV	2000 [0.6],	3500 [0.6],	5000 [0.6],	6500 [0.6]
ADV BGT	50 [0.6],	600 [0.6],	1150 [0.6],	1700 [0.6]

As described in our methodology, these possibilities are converted into linguistic form. The final linguistic outputs are as follows.

Variable Name	Value	Linguistic Output
DIPC	522.2	Low
MARKET SIZE	4441.2	Medium
PRICE	4.85	Very Low
AFCF	0.1625	Somewhat Medium
DEMAND	1000	High
SALES REVENUE	4250	Somewhat high
ADVT. BUDGET	875	High

## 5. Conclusion

We have presented a fuzzy sets based technique for qualitative analysis of feedback loops. It is a better solution compared to traditional system dynamics approach. Since we use computers in decision making, we have to follow certain precise framework to perform computations. At the same time, everything cannot be precisely defined at managerial level. This means uncertainty is there in the system. Fuzzy sets help us in capturing this uncertainty. We have developed complete methodology and demonstrated the same. The success of our methodology depends on the fact that how much correct information is fed in to our system. Once that information from a suitable expert is gathered qualitatively our methodology will generate an expert system which can be used a number of times for analysis. The beauty of our approach is that we are accepting qualitative inputs and giving qualitative outputs. This feature will help in wider acceptability of our proposed methodology.

## References

- [1] Zadeh, L.A., "Fuzzy sets", *Information and control*, Vol. 8, No. 3, pp. 338-353, 1965.
- [2] Kaufmann A. and Gupta M.M. (1988), *Fuzzy Mathematical Models in Engineering and Management Science*, Elsevier, ISBN: 0444705015.
- [3] Zadeh L.A. (1986), Test score semantics as a basis for a computational approach to the representation of meaning, *Literacy and Linguistic Computing*, vol. 1, issue 5, pages: 24-36.
- [4] Stermann, J.D., "*Business Dynamics: System Thinking and Modeling for a Complex world*", McGraw, 2000.
- [5] Jeng, B., Chen, J.X., and Liang, T.P., "Applying data mining to learn system dynamics in a biological model", *International Journal of Expert Systems with Applications*, Vol. 30, No.1, pp. 50-58, 2006.

- [6] Hjorth, P., and Bagheri, A., "Navigating towards sustainable development: A system dynamics approach", *International Journal of Futures*, Vol. 38, No. 1, pp. 74-92, 2006.
- [7] Yim, N-H., Kim, S-H., Kim, H-W., and Kwahk, K-Y., "Knowledge based decision making on higher level strategic concern: system dynamic approach", *International Journal of Expert Systems with Applications*, Vol. 27, No. 1, pp. 143-158, 2004.
- [8] Al-Najjar, B., and Alsyouf, I., "Selecting the most efficient maintenance approach using fuzzy multiple criteria decision making", *International Journal of Production economics*, Vol. 84, No. 1, pp. 85-100, 2003.
- [9] Tahera, K., Ibrahim, R.N., and Lochert, P.B., "A fuzzy logic approach for dealing with qualitative quality characteristics of a process", *International Journal of Expert Systems with Applications*, Vol. 34, No. 4, pp. 2630-2638, 2008.
- [10] Polat, S., and Bozdağ, C.E., "Comparison of fuzzy and crisp systems via system dynamics simulation", *European Journal of Operational Research*, Vol. 138, No. 1, pp. 178-190, 2002.
- [11] Karavezyris, V., Timpe, K.P., and Marzi, R., "Application of system dynamics and fuzzy logic to forecasting of municipal solid waste", *International Journal of Mathematics and Computers in Simulation*, Vol. 60, No. 3-5, pp. 149-158, 2002.
- [12] Forrester, J.W., "*Industrial Dynamics*", John Wiley & Sons, 1961.
- [13] Coyle, R.G., "*System Dynamics Modeling*", Chapman & Hall: London, 1996.
- [14] Tsoukalas, L., and Uhrig, R., "*Fuzzy and Neural Applications in Engineering*", John Wiley, 1997.
- [15] Jang, J.S.R., and., Sun, C.T., "*Neuro-Fuzzy and Soft computing: A Computational Approach to Learning and Machine Intelligence*", Prentice Hall, 1997.
- [16] Mamdani, E.H., and Assilian, S., "An experiment in linguistic synthesis with a fuzzy logic controller", *International Journal of Man-Machine Studies*, Vol. 7, No. 1, pp. 1-13, 1975.
- [17] Takagi, T., and Sugeno, M., "Fuzzy identification of systems and its applications to modeling and control", *IEEE Transactions of System. Man Cybernetics*, Vol. 15, No. 1, pp. 116-132, 1985.
- [18] Manoj Kumar Jain, "*Development of an Expert System Generator using Fuzzy Sets*", International Conference on Management, INCOM 2008, 22-24 September, 2008, Udaipur, India. Edited Book: "Managing Global Competition- A Holistic Approach", Macmillan Publication. ISBN 10: 0230-63646-2 and ISBN 13: 978-0230-63646-0. Pages: 509-517.
- [19] Manoj Kumar Jain: "*An Efficient Expert System Generator*", Aishwarya Research Communication, Special issue Multidisciplinary International Conference: Emerging Issues and Global Economic Senario, Focus: Emerging Trends in ICT (August 21-24, 2009), ISSN: 0975-3613, Vol. 1, Part 2, March 2010. Pages: 105-110.
- [20] Manoj Kumar Jain: "*Information and Communication Technology Expansion using Fuzzy Sets*", Aishwarya Research Communication, Special issue National Conference: Educational Reforms: Insights and Analysis (October 1-3, 2010), ISSN: 0975-3613, Vol. 2, Part 1, October 2010. Pages: 79-85.