

Searching the Arcane Origins of Fuzzy Logic

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*To the great Japanese people, with my more sincere condolences,
after his terrible earthquake with later tsunami, and nuclear problem.*

Abstract

It is well-known that Artificial Intelligence requires Logic. But its Classical version shows too many insufficiencies. So, it is very necessary to introduce more sophisticated tools, as may be Fuzzy Logic, Modal Logic, Non-Monotonic Logic, and so on. When you are searching the possible precedent of such new ideas, we may found that they are not totally new, because some ancient thinkers have suggested many centuries ago similar concepts, certainly without adequate mathematical formulation, but in the same line: against the dogmatism and the dualistic vision of the world: absolutely true vs. absolutely false, black vs. white, good or bad by nature, 0 vs.1, etc. We attempt to analyze here some of these greatly unexplored, and very interesting early origins.

Keywords: History of the Science; Logics; Artificial Intelligence; Ancient Greek Philosophy.

1. Justification

Each time that we write the first line of a new paper, we have a great suspicion: Is it really necessary? The answer to this question may be dual: their theoretical and practical basis and potential consequences. In this case, the topic is a very old friend, because from many years ago, I return to the idea of the remote origins of the Fuzzy Logic, Non-monotonic Logic, and in general, of this new and promising field of current and future Science.

When I travel this month (March 2011) to the University of Cordova, in Spain, to participate in a very interesting Seminary on “*El Pensamiento Escéptico en la Historia*” as general topic, organized by the “*Sociedad Ibérica de Pensamiento Griego*”, this activate in our mind this old question, and jointly, if the Skeptical current is connected with a primitive thought in the line of Fuzzy Logic, or the very curious coincidence (?) according many people that belonged to this philosophical tendency were physicians. Why? I attempt to give plausible answer to some of these very interesting questions.

2. Short history of Fuzzy Logic

We may to think that Fuzzy Logic would be a recent field, where the people are working from very few time. But their origins proceeds surely from the Old Greek philosophers and scientists, in particular, from *Plato* (428-347 B.C.), and its revered disciple, *Aristotle* (384-322 B.C.). Possibly they are the first considering which the things are not of uniquely one way or its opposite, but there exists all a range of greases, i.e. that there are a complete intermediate scale. Because both are pioneers suggesting the existence of many different degrees between trueness and falseness.

From such initial thinkers we may to mention two essential names, *David Hume* (1711-1776), and *Immanuel Kant* (1724-1804). They will be obtains a detailed analysis of such concepts. Both philosophers conclude that both reasoning and knowledge are reached through the experiences of our lives.

In fact, David Hume choose the Common-sense Logic. Kant possesses the opinion that only mathematicians can provide very clear definitions. Detecting some contradictory principles on Classic (also named either Formal or Mathematical) Logic.

So, at first of 20th century the British mathematician *Bertrand Russell* spread the idea that the Mathematical Logic reveals inherent contradictions. He produces an extensive study on the vagueness of the language, concluding that the vagueness will be a degree.

Also may be considered *Georg Cantor* (1845-1918), and its crisp or classic set theory, being precursor in some sense of Zadeh's fuzzy set theory. And *Charles Sanders Peirce* (1839-1914) by its Uncertainty Theory.

The Austrian thinker *Ludwig Wittgenstein* (1889-1951) analyze the different meaning (senses) of a word. So, L. Wittgenstein observe that on a language the same word can express different moods and manners.

The Polish logician *Jan Lukasiewicz* (1878-1955) developed the first Vagueness Logic; in fact, a three-valued logic, with three values, 0, $\frac{1}{2}$ and 1, corresponding to *False, Possible and Truth*. More later he generalizes to four-valued, and many-valued logic (MVL, by acronym). Therefore, he introduces the membership degrees of each element to each corresponding set. So, we dispose of a membership function of which range cover the unit interval of the real line, $I = [0, 1] \subset \mathbf{R}$.

The father of the new formalization of the term "fuzzy", jointly with the Fuzzy Logic and Fuzzy Set associated theories was *Lofti Asier Zadeh*, which in 1965 publish its famous and seminal paper, the so-called "*Fuzzy Sets*" [13-16].

Such ideas collect the precedent work of many authors of different disciplines. Between them, we must to mention *Max Black* (1909-1988), philosopher and also quantum physicist, which in 1937 propose the necessity of a theory of "vague sets".

Also the more than notable British mathematician *Bertrand Russell* (1872-1970), considered a great precursor, searching logical contradictions and their plausible solutions.

Either the German quantum physicist Werner Heisenberg, with its famous "*Uncertainty Principle*".

Or the very important contribution of the Polish mathematician Jan Lukasiewicz (1878-1955), which formalizes many of the more important technical questions.

All them contributed to produce the magma from which derives the famous papers of Lofti A. Zadeh, not only the aforementioned [13] "Fuzzy Sets" (1965), but also on 'Fuzzy Algorithm', three years afterwards [14].

At beginning (as almost all the new ideas) the papers of Lofti A. Zadeh not obtain very good scientific reception, but with the past of the time, they obtain more and in many cases inspired continuators. So such theories were considerably extended, and accepted into the very professional scientists. The initial resistance against the Fuzzy Logic may be very surprising, but certainly when Zadeh visiting IBM to expose their ideas, IBM says that they are not interested! It is, in fact, a memorable example of commercial occidental vision.

The purpose, or objective, of Lofti A. Zadeh was to create a formalism to manage efficiently the natural imprecision of human reasoning. In 1971 it appears its paper so-called "Quantitative Fuzzy Semantics", where appears many formal elements of Fuzzy Logic, and their methodological aspects, with derivations that until today are frequently used on applications. From 1973, with the basic theory of Lofti A. Zadeh on fuzzy controllers, many researchers initiate the application of Fuzzy Logic to control diverse processes.

It appears many different groups of researchers on Fuzzy Logic insome Little Japanese universities. So, the profesors Terano and Shibata in Tokio, or the profesors Tanaka and Asai in Osaka, producing important contributions, and simultaneously developing Fuzzy Logic theory and their applications. And parallel to the development of impressive applications of Fuzzy Logic, *Takagi y Sugeno* to explain the first approximation to construct *Fuzzy Rules*, from learning data (*Fuzzy Learning*).

Other decisive factor to continue with an increasing research on this field are the Neural Nets (or Networks; NNs, by acronym) and its similitude with the *Fuzzy Systems*. By searching mutual interrelationships between both useful techniques, we may obtain the *Neuro-Fuzzy Systems*, using learning methods based on Neural Networks to identify and optimize their parameters. To

finish this introductory historical note, then appears the *Genetic Algorithms*, that jointly with Neural Networks and Fuzzy Systems are very powerful tools, and so they are indeed of increasing interest on actual and future mathematical investigations.

3. Non-monotonic Logic

We call *Non-monotonic logic* to a logical system where the logical system the logical consequence relation is non-monotonic.

Many of the logical systems [3, 6], as for instance Mathematics, have a logical consequence relation which are monotonic. Because add some formula to a theory this never produce a reduction of the set of consequences.

Symbolically this can be expressed by

$$\text{If } F \rightarrow A, \text{ then } F \cup G \rightarrow A, \forall G$$

Being A an arbitrary formula, with F and G both set of formulas, and symbolizing \rightarrow the logical consequence. I. e.

If $S \rightarrow C$ and ever $S \cup T \rightarrow C$, for any S and T, sets of valid propositions, then this logic is of monotonic type.

Instead of this,

If $\exists T$ such that $S \cup T \not\rightarrow C$, then it will be a non-monotonic logic.

We may note a crucial point: we don't modifies here our disposable base of affirmations or base of facts (by AI terminology), because if we do this, it would be evident the possibility of modify the set of consequences. But instead of this, our argumentation departs to maintain the same elements on the original knowledge base, only now enlarged with new propositions, or new information. This occurs very frequently on Medicine, where continuously the new discoveries invalid old beliefs. So, according to the knowledge goes advancing, this which is now prescribed, it will be tomorrow forbidden. And this produces the necessity of go quickly out of the Aristotelian or Formal Logic, where the reasoning is of monotonic type, as on the Mathematics. This description shows the mechanism of the non-monotonic reasoning, with a totally provisional and ever revisable character, evolving with the time; the opposite of Mathematics: the same concluding forever.

4. Rule-Based Systems

The Rules shows a great advantage on the Classical Logic [6]. Usually, the Rules must appear grouped, as a system of Rules, each of them with its antecedent and consequent. They have two procedures of firing, according which we apply chaining forward or chaining backward. It is because in each system we can proceeds from the respective antecedents to their consequents, or vice versa, until to reach the goal fact (or "concepto-meta", in Spanish), i.e. the desired last fact.

In the Classical Logic the Reasoning was *Monotonic*, with inferences without contradiction with the pre-existing, in SBR. Nevertheless in the Rule-Based Systems (RBS, by acronym), we may delete facts or affirmations of the Base of Facts, according the new inferences. This makes the Reasoning Non-Monotonic, because we can modify the conclusion. Then, appear a crucial question: which we must to make with the conclusion of the affirmation now invalidated?

For this problem [3], we need to introduce the concept of *Type of Dependence of a Rule*, which can be *Reversible*, if we delete the affirmations, then we delete automatically the above inferred facts; or *Irreversible*, if the facts, once inferred, will be not deleted neither changed. And in the case of some applicable rules at time, which must be executed firstly? Such Rules constitutes, in each step, the *Conflict Set* (obviously, a dynamic set). The subjacent decision problem is called *Resolution of Conflicts or Control of Reasoning*.

5. More on the remote origins of Fuzzy Logic

Apart of the aforementioned Greek philosophers (Plato and Aristotle), we must to observe other interesting names [7], as either:

- *Heraclitus of Ephesus*, and Anaxagoras, both deniers of the Non-contradiction Principle, according to Aristotelian testimony. Also may be interesting Protagoras, on this Principle.

Either the skeptical school or tendency, with:

- *Pyrrhon of Elis* (c. 360-270 B.C.), its human model; he never writes their teachings;
- *Timon of Fliunte*, its disciple (320-220 B.C.); he write a compilation of the Pyrrhon ideas;
- *Carneades (214-129 B.C.) and Arcesilao (315-241 B.C.)*. Both belongs to the Platonic school, and so, to the Academic Skepticism; etc.

Also their successors on this way of thought, as:

- *Sixtus Empiricus* (2.nd or , or first century A.C.), a physician and writer, whose works on Pyrrhonism are very interesting [8-11];
- *Diogenes Laertius* [2], with its great book *The Lives and Opinions of Great Philosophers*. The analysis of Pyrrhon belongs to its Vol. IX;

Both, Sixtus Empiricus and Diogenes Laertius, were Roman and interesting writers.

This current remains during the time, as apparently missing, and reappears with modern thinkers as either:

- the Spanish physician *Francisco Sanchez*, “*el Escéptico*”, author of *Que nada se sabe (Quod Nihil Scitur, 1581)*; W. Willeband consider to Sanchez at the same level as I. Kant; and it was also greatly considered by Leibnitz; or the very good French writer (father of the *Essays*, a new and very productive literary vehicle, similar in some aspects to modern papers):
- *Michel de Montaigne (1533-1592)*.

But many others (some of them more or less dubious skeptical, but influenced by this tendency), as:

- *Pierre Bayle (1647-1706)*
- *David Hume (1711-1776)*

Possibly, we may detect its influence on renowned great thinkers, as:

- René Descartes was deeply influenced by Francisco Sanchez on its *Discourse de la Méthode*.

The first lines of the book of Sanchez remember clearly to Descartes later book, and also on many other passages:

*Es innato al hombre querer saber; a pocos les fue concedido saber querer;
a menos, saber. Y a mí no me cupo suerte distinta a la de los demás.*

- Immanuel Kant; for instance, on its *Goldberg Logik*.

6. More Historical Notes

Some notes about Pyrrhonism. Its founder was Pyrrhon of Elis (360-270 BC.), contemporary of Alexander the Great. It participates with the expeditionary forces of this conqueror until the Indian territories, where he saw in the fakirs an admirable example of happiness, only reached from the indifference to circumstances [2, 8-11]. It is very notable the influence produced on Pyrrhon by the gymno-sophistes.

Believing that equal arguments can be offered of both sides of any proposition, he dismissed the search for truth as a vain endeavor. This opened the possibility of the admittance of degrees of truth, possibly variables, or it appear as very next to them.

Pyrrhon [7] was a pupil of Anaxarchus of Abdera. For many people, the Skepticism consist more in either a tendency or cultural trend than in a very school. Indeed it appears as a generalized admiration into the initial group, and also the subsequent tradition, seeing to Pyrrhon as an authentic model of wise man [8-11].

Some logical reflections. Aristotle established their *Laws of Thought*:

- *Identity Principle*
- *Principle of Contradiction*
- *Law of Excluded Middle*

They have conditioned -and in some cases obstructed- the development of Logic in Western culture [3, 4]. This fact is less oppressive in Oriental countries (China, Japan, Korea, India, etc.), because the Buddhist tradition admits with naturalness, on the things and thoughts, the possibility of simultaneously be and not to be, and that the propositions may be both, true and not true by the

assignation of a gradation. This connects directly with the Fuzzy Logic and the Theory of Fuzzy Sets, who introduced the membership function, which range to move into the unitary real interval, denoted by $I = [0, 1] \subset \mathbf{R}$.

In Ancient Greek some philosophers have pronounced affirmations that are compatible with a new way of think [2]. So Plato in their *Dialogues* makes a good compilation of thoughts of Socrates, his teacher, and other precedent thinkers.

As aforementioned, and according to Aristotles, Heraclitus of Abdera and Anaxagoras both denied the Non-contradiction Principle, according to it is not possible to reach simultaneously p and $\neg p$. Also may be interesting to consult Protagoras about this Principle.

The psychological version of such Principle of Non-contradiction may be this: Nobody can to think “ p ” and “ $no p$ ”.

On modern formulation, the collection of Fuzzy Sets, with the operations of union, intersection and path to the complementary defined by Zadeh, will be not a Boolean Algebra. So, it is fully different to the now classical theory of George Cantor, about the Crisp Sets. And it is because the violation of both principles, Non-contradiction and Third Excluded. Nevertheless, by the modified definitions of the Polish mathematician Jan Lukasiewicz, they are fulfilled [3, 4].

7. Some of Oriental Thought

Searching on the old Oriental doctrines [5], we can found some tendencies of Buddhism very related with the degrees of truth, and also the two-truths doctrine [1a].

Nāgārjuna (150-250 A. C.) was the Indian founder of the Madhyamaka school of the Buddhism Mahayana. The term Madhyamaka signifies “Middle Way, or Via Media”. It was the more prominent philosopher on the Indian tradition.

Nāgārjuna was also a practitioner of traditional Indian Ayurvedic medicine. Between his conceptualizations, we can found some descriptions of the circulatory system and blood tissue. In Western medicine, at least until now, Ayurvedic medicine is considered alternative, as merely a complementary treatment, but never replacing the traditional diagnosis and “farmacopea” of Western expert. Balance is emphasized; suppressing natural urges is seen to be unhealthy, and doing so may almost certainly lead to illness. So, the people are cautioned to stay within the limits of reasonable balance and measure.

His primary contribution is in the use of the concept of "emptiness" which brings together other key Buddhist doctrines, to refute the metaphysics.

For *Nāgārjuna* [1a], as for the Buddha in the early texts, it is not merely sentient beings that are "selfless" or non-substantial; all phenomena are without any "self-nature", and thus without any underlying essence. They are empty of being independently existent. This is so because all things arise always dependently: not by their own power, but by depending on conditions leading to their coming into existence, as opposed to being.

Nāgārjuna was also instrumental in the development of the two-truths doctrine, which claims that there are two levels of truth, one which is directly true, and one which is only conventionally or instrumentally true.

So, “*this world is supported by a polarity, that of existence and non-existence... Everything exists: That is one extreme. Everything doesn't exist: That is a second extreme. Avoiding these two extremes, the Tathagata teaches the Dhamma via the middle...*”.

Nāgārjuna differentiates between conventionally true and ultimately true teachings, but he never declares any conceptually formulated doctrines to fall in this latter category.

This was famously rendered in his *Tetralemma*, with the logical propositions:

X (affirmation)
non-X (negation)
X and non-X (both)
neither X nor non-X (neither)

So, we may establish three original traditions of Logic: Greek, Chinese and Indian traditions. This latter continued until very modern times, through the Navya-Nyaya school of logic.

Because the *Rigveda* contains some speculatively considerations in terms of logical divisions. More exactly: “A”, “not A”, “A and not A”, and “not A and not not A”, which coincides with the above Tetralemma.

Panini, in the 5th. century B.C., develop a logic that have certain interesting similarities with Boolean logic.

The Navya-Nyaya school developed a conceptual scheme that allowed solve problems in Logic, by developing theories very similar to Gottlob Frege’s ideas.

All these schools are discovered by British scholars in the late 18th century. We mention to H. T. Colebrooke’s analysis of inference [1b], by comparison with Aristotelian logic, resulting that the Aristotelian syllogism could not account for the Indian syllogism.

Hermann Weyl, in 1924, wrote that “Occident mathematics has in the past centuries broken away from the Greek view, and followed a course which seems to have originated in India, and which has been transmitted, with additions, to us by the Arabs; in it the concept of number appears as logically prior to the concepts of geometry”.

In fact, Indian logic [1a] has deeply influenced on many of modern logicians, as Charles Babbage, George Boole, or Augustus De Morgan. They belong to 19th. century, but may be named “modern” in many senses. The same De Morgan says that “the two races which have founded the Mathematics, those of the Sanscrit and Greek languages, have been the two which have independently formed systems of logic”.

Therefore, mathematicians are finally aware of the notable influence of Indian Mathematics/Logic on the European Mathematics/Logic.

8. The last but not the least question

When we participate in the aforementioned Scientific Seminary (on Skepticism, at Cordoba University), the Brainstorming (Debate) after the cycle of conferences produces some interesting questions.

One of the more curious for me was the coincidence into the thinkers of skeptical tendency: almost all belongs to the Physician profession. Why? It is the question.

Already this question was expressed by Sextus Empiricus on his *Hypotyposis Pyrrhonicae* (I, 236-238): “some people said that the Skeptic philosophy is identical to the empirical current” (empirical as current usual between physician professionals). But he says that are would be not identical [12].

My answer-hypothesis consists to suggest that the reason may be the own type of reasoning connatural with Medicine: it uses Non-monotonic logic, as the above described. And given the mutual relationships between Skepticism and this way to handle the knowledge, it appears as a plausible justification of this fact.

A notable coincidence is related to the great Buddhist thinker Nāgārjuna, whose professional practice was precisely the traditional Indian Ayurvedic medicine, and many others cultivating the Buddhist world vision. Also many other of such Indian philosophers and wise men are Ayurvedic physicians [1b].

Recall that *Ayurveda* is grounded in the physics of the “five elements”; concretely, they will be air, fire, water, earth and ether, all them composing the Universe, including the human body. Ayurveda also focuses on exercise, massage, yoga, and meditation. Natural Medicine, may be, but Medicine.

So, these new coincidences support the precedent plausible explanation about this interesting and recursively observable phenomena. Because we can observe a great agreement between Skepticism, their Indian origins, and the physician logical procedures, by the application of Non-monotonic logic.

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