How much Mathematics does Economy Need? ...Or... Some Brief Epistemological Excursions into the Mathematics located on the border with Social Sciences

Tiberiu Socaciu
“Ştefan cel Mare” University of Suceava, Faculty of Economics, Romania,
“Vasile Goldis” West University of Arad, Faculty of Informatics, Romania
“Babes-Bolyai” University of Cluj-Napoca, Faculty of Economics, Romania
socaciu@inf.ro

Abstract
In our paper we try to answer to the epistemological question how much mathematics does economy need.

Keywords: economics, economy, mathematics, epistemology

1. I am a mathematician!
Norbert Wiener1 is2 probably the most famous case3 who autobiographically claimed “I am a mathematician”. There are also other examples4 less assertively stating that “I want to be a mathematician”, although with arguments. I can only apologize for this attempt with obvious epistemological facets bearing the notice: I am a mathematician!

2. What is a mathematician?
In Roman Antiquity, mathematician was almost perfectly synonymous with astrologer6. Only that, over the centuries, the meaning of “mathematician” has evolved. Thus, nowadays, we attribute the word “mathematician” to somebody who is in at least one of the following cases:
1) knows how to “juggle” with certain types of “mathematical objects”;
2) has certain above-average skills in the subject called “Mathematics”;
3) is the holder of a diploma certifying this title.
I am a mathematician in terms of the third point of view.

3. Is Mathematics a “Small Creation”?
Although slightly esoteric in origin, there are7 two concepts which can be summarized as:
1) Small Art, or Small Master;
2) Ars Magna, or Great Master.
And since the Small Art is just a stage in achieving the Great Art, we could say that any science has to admit that Mathematics is like a Small Art if this, in its turn, regards itself as the Great Art. But this seems to be a tautology, because any science that pretends to have reached maturity has to be a small Great Art!

4. Do Social Sciences need Mathematics?
Undeclared epistemologists of Mathematics8 claimed that some representatives of Social sciences need Mathematics only to make their discipline look like “science”. Some teachers use mathematical tools for balance, perhaps to impress course attendants. Could this be a slight analogy

---

1 Norbert Wiener (1894-1964), American mathematician. Wiener is known as the father of Cybernetics (see the fundamental work [Wiener1948], the second edition translated into Romanian [Wiener1966]).
2 See [Wiener1972] or one of the English editions [Wiener1981].
3 According to [Marcus2008], p. 1, paragraph A fi matematician.
4 See [Halmos1988].
5 See [Hutin1992], p. 37.
6 See [Hutin1992], p. 84.
7 See [Marcus1987].
to the teacher in Ionescu’s Lectia9? Are we indeed approaching the hyperbole of killing the students? There is a theory according to which mathematical sciences are somewhere inside10 a triangle whose points are:

1) positive sciences;
2) descriptive sciences;
3) affective sciences.

Probably a more precise localization of Social sciences within this triangular quadrinomial would highlight a possible connection to mathematical sciences. Mathematics has been reproached11 that it would dodge the specificity of social sciences in its relationship with these, treating them similarly to the sciences of nature or, sometimes, to engineering sciences. Basically, almost everything that does not suit a certain last is viewed nonchalantly as the area of applied Mathematics... The analysis of a list of Nobel Prizes in Economic Sciences12 will generate a surprise: most awards have been made for mathematical models in the area of Economy. I have attempted to study the sites of several universities that offer bachelor degrees in Economics: I looked at the subjects and I was horrified! There are faculties with 1 (one) course in Mathematics for 1 (one) semester and 1 (one) course in Informatics of 1 (one) semester. Therefore, a total of 2 (two) courses! Only hardly have I found 2 (two) courses in Computer Science for some programs where it is regarded as a “speciality” subject13, reaching a number of 3 (three) courses, the exception seeming to be the program of Economic Informatics /Economic Cybernetics. I have also looked at the subjects studied by the students of a famous American school14, which promotes all courses as open-courses15. On every occasion, I recommend the study of this site16 to my students but also to my colleagues. Why?

5. But why shouldn’t we ask ourselves How much Physics does Economy need? What is, in fact, Econophysics?

There is a component of Finance called Continuous-time Finance. The aim of this sector seems to be the study of anything that can be modeled by continuous stochastic processes. The assessment of financial assets and financial derivatives started by Bachelier17 via Samuelson18 and the famous triad Black19-Scholes20-Merton21 has developed this area based on some analogies from Physics22. Everything that has been built by Statistical Physics from Avogadro’s number23, the

---

9 See [Marcus1987], p. 11, [Ionesco2003].
10 Probably very close to the barycenter.
11 See [Marcus1988], p. 255.
12 See [***2].
13 For example ATI – Accounting and Trust Informatics.
14 Sloan School of Management, functioning within the M.I.T.
15 The accepted terminology is OCW – Open Courseware.
16 See [***3].
17 Louis Jean-Baptiste Alphonse Bachelier (1870–1946) was a French mathematician. He is considered the first to have developed the model of Brownian movement in his doctoral dissertation, even before Einstein. Bachelier is considered a pioneer in financial mathematics and stochastic processes.
18 Paul Anthony Samuelson (b. 1915) is an American economist who was awarded the Nobel Prize in Economics in 1970.
19 Fischer Sheffey Black (1938–1995) was an American economist, with a doctorship in Applied Mathematics. Author, together with Scholes, of the famous Black-Scholes equation. In 2002, American Finance Association has set up an award called Fischer Black to be awarded biannually.
20 Myron Samuel Scholes (b. 1941) is an American economist who, together with Robert Cox Merton, was awarded the Nobel Prize in Economics in 1997. He is the author of the famous Black–Scholes equation, together with Fischer Sheffey Black.
21 Robert Cox Merton (b. 1944) is an American economist, laureate of the Nobel Prize in Economics, together with Myron Samuel Scholes, in 1997.
22 More precisely, thermodynamics.
23 Lorenzo Romano Amedeo Carlo Avogadro, Count of Quaregna and Cerreto (1776–1856, Turin) was an Italian physicist with studies in the legal field, too. He taught Mathematics and Physics at the Royal College in Vercelli. He set
Central limit theorem, Gauss’s distribution\textsuperscript{24}, to Maxwell’s laws\textsuperscript{25} can be recovered in modeling the behaviour of the price of an asset on the market. Let us not forget that the Black-Scholes\textsuperscript{26} equation is equivalent\textsuperscript{27} to the equation of heat\textsuperscript{28}, which makes that the know-how related to numerical solving (for example by Crank\textsuperscript{29}-Nicolson\textsuperscript{30} schemes eventually adjusted with Hyman-Kaplan schemes\textsuperscript{31}) or qualitative aspects (for example, Green’s functions\textsuperscript{32}) be inherited relatively easily. Perhaps it is not strange that the greatest developers of Continuous-time Finance are mostly physicists\textsuperscript{33}... And last but not least, I find pleasure in thinking about my recent concerns\textsuperscript{34} in the area of probability distributions, the BHP distribution\textsuperscript{35} that was built starting from a problem of magnetization in two dimensions\textsuperscript{36}. Gonçalves and Pinto have succeeded in demonstrating the universality of this distribution in problems of hydrology, astrophysics and even stock exchange\textsuperscript{37}. And all these we see in one of the most famous children of Mathematics and physics: Mathematical physics... Are we close to what some call Econophysics?

6. Is Financial Engineering a part of Mathematics, Physics, Engineering or Economy? Or is it nothing of all these and Financial engineering MUST remain an esoteric matter, available only to the initiated?

In recent years, a new discipline called Financial engineering\textsuperscript{38}, has appeared in the world of Economic sciences. I shall not go into details concerning the description of the discipline’s

up the first department of Mathematical Physics in Turin. In 1811, while teaching at Vercelli, he issued the so-called molecular hypothesis, according to which two equal volumes of any gases, existing within the same pressure and temperature, contain the same number of molecules. This number of molecules is known as Avogadro’s number or the Avogadro constant.\textsuperscript{24}

\textsuperscript{24} Carl Friedrich Gauß, Carolo Friderico Gauss in Latin, (1777–1855) was a famous German mathematician, physicist and astronomer. Gauss’s writings (404, only 178 published) concern several domains, from disciplines of Mathematics, Physics to Geodesy or Astronomy.

\textsuperscript{25} James Clerk Maxwell (1831–1879) was a Scottish physicist, author of a set of equations describing the basic laws of electrotechnics. Maxwell’s equations prove that electricity, magnetism and light are manifestations of the same phenomenon: the electromagnetic field. He discovered Maxwell’s distribution in the area of the molecular kinetics of gases. He is famous for the first coloured photograph taken in 1861.

\textsuperscript{26} See [Black1973].

\textsuperscript{27} See [Socaciu2009a], [Fărcaş2006], p. 226, with the notice that the formula for the $\beta$ parameter is wrong and [Wilmott2007], p. 160.

\textsuperscript{28} We have provided a demonstration of the equivalence between the BS equation and the equation of the diffusion in [Socaciu2009a].

\textsuperscript{29} John Crank (1916–2006) was a British mathematical physicist, best known for his work in the field of differential equations with partial derivatives (the equations of Mathematical physics). During the Second World War he worked on ballistics. Together with Nicolson, he worked on the numerical solution to the heat equation, developing the Crank-Nicolson schemes.

\textsuperscript{30} Phyllis Nicolson (1917–1968) was a British mathematician best known for her collaboration with John Crank in the field of the discretization of equations with partial derivatives, which led to the Crank-Nicolson schemes.

\textsuperscript{31} See [Racoveanu1977], p. 54.

\textsuperscript{32} George Green (1793–1841) was a British mathematician and physicist. He introduced significant concepts such as Green’s theorem, the idea of potential functions, George Green’s functions. Green was the first who mathematically explained electricity and magnetism, opening the path for other scientists such as James Clerk Maxwell, William Thomson and others. The potential theory was developed in parallel with Gauss.

\textsuperscript{33} Obviously, there is an approach to these areas by way of discrete stochastic processes that are nothing else than the famous time series. Here, one can relatively easily interweave predictabilities in the area of artificial intelligence (neuronal networks, genetic algorithms, data mining), fractals (how easily can one reach chaos, for it takes only 4 iterations at the most, according to the theory of bifurcation!) or tools from the Modern Econometrics (self-regressive models or others of the same class).

\textsuperscript{34} See [Socaciu2009b], [Socaciu2009c], [Socaciu2009d].

\textsuperscript{35} Named after its “inventors”: Bramwell-Holdsworth-Pinton.

\textsuperscript{36} See [Bramwell1988].

\textsuperscript{37} See [Gonçalves2009].

\textsuperscript{38} Unfortunately, the post-December signification has been spoiled in Romanian.
object, but I will only mention the scientific exceptional approach\(^{39}\). Mathematicians, physicists and traders are working together to incorporate the use of certain mathematical advanced concepts\(^{40}\), as well as computational technologies \(^{41}\). And since there is always a stake, the computational effort can be immediately applied in this area, bringing closer to Economy another child of Mathematics: Informatics. But why so little worldly interest\(^{42}\) for this discipline? Is it something deliberate? I can only note with pleasure one of the very few books issued in Romania concerning this topic, whose first author is the CEO of SIF Banat-Crișana\(^{43}\).

7. On the border between Economy and Mathematics: an academician’s viewpoint

In his investiture speech at the Romanian Academy, academician Solomon Marcus said that\(^{44}\): “The close connections between Economy and Mathematics are several centuries old. During the 20\(^{th}\) century and especially during its later half, mathematical language became the prevailing means of expression in economic phenomena, a fact reflected in the many Nobel prizes awarded to some highly mathematized works. This fact is not aloof from the appearance and development of the theory of strategy games, starring John von Neumann\(^{45}\), Oskar Morgenstern\(^{46}\) and John Nash\(^{47,\ast}\).

8. What is economy or who are the “half-breeds” of Economy?

We know\(^{48}\) that Economic science is difficult to define … It has gradually turned from Economic science into the system of economic sciences which can be found in the following classification\(^{49}\):

1) Fundamental economic sciences – Political Economics, Micro Economics, Macro Economics;
2) Functional economic sciences – Statistics, Accounting, Management, Finances etc;
3) Concrete economic sciences – Industrial Economics, Economics of Agriculture, Economics of Education etc;
4) Historical economic sciences – Economic Doctrines, History of the national economy, History of global economy;
5) Border economic sciences – Economic Geography, Econometrics, Economic Cybernetics, etc.

We should find, on the border of the ensemble of economic sciences, many economic half-breeds\(^{50}\), among which we will certainly find Economic Mathematics, Economic Cybernetics, Classic Econometrics (based on models of linear, log-linear and logistic regression) and Modern Econometrics (non-linear regressions, time series, stochastic processes, data mining, neuronal networks, genetic algorithms), Economic Informatics, Financial Mathematics, Actuarial Mathematics, Financial Engineering, Operational research, Mathematical statistics (with the four

\(^{39}\) See [Altăr\(2002\)] and [Altăr\(2003\)], two university courses on this subject.

\(^{40}\) For example, the Ito differential calculus, stochastic Wiener-Bachelier-Einstein processes, white noises etc.

\(^{41}\) See [Kim\(2000\)], [Stix\(1998\)].

\(^{42}\) In terms of the qualified study of this discipline at the level of bachelor or master degrees.

\(^{43}\) See [Farcas\(2006\)].

\(^{44}\) See [Marcus\(2008\)], paragraph Mathematics, from tool to language /

\(^{45}\) John von Neumann, Margittai Neumann János Lajos in Hungarian, (1903-1957) was an American mathematician of Hungarian origin who made significant contributions to quantum physics, functional analysis, set theory, topology, economy, computer science, numerical analysis, hydrodynamics of explosions, statistics and many other fields of Mathematics, being one of the most important mathematicians in history.

\(^{46}\) Oskar Morgenstern (1902 –1977) was a German economist, born in Austria. Together with John von Neumann, he developed the game theory, the expected utility hypothesis etc.

\(^{47}\) John Forbes Nash, Jr. (b. 1928) is an American economist and mathematician who contributed to the game theory, differential geometry, partial differential equations, complex systems. In 1994, he, Reinhard Selten and John Harsanyi were awarded the Nobel prize.

\(^{48}\) See [Pohoata\(1984\)], p. XX-XX.

\(^{49}\) According to [***1].

\(^{50}\) A term invented ad hoc by the undersigned. I hope it will not grate the ears of specialists!
classic components: selection theory, estimation theory, checking statistical hypothesis, statistical decision theory) and its sister, Economic statistics, Decision Theory and many others. What do all these children share?

9. Half-breeds are always successful children!

Concerning the half-breeds of Mathematics, we cannot leave aside the Mathematical Physics, the oldest half-bred of Mathematics. But, there subsequently occurred Mathematical Chemistry, Mathematical Biology, Mathematical Psychology, Mathematical Economy, Mathematical Linguistics... Perhaps the fattest child of Mathematics is Informatics who, coming of age, is trying to make its own family: Informatic Mathematics, Informatic Physics, Informatic Biology, Economic Informatics... sometimes I sit and wonder, or ask others: What is Economic Informatics? It is a branch of Informatics that has entered the field of economic sciences? Or is it the other way round? Are Financial Mathematics the same with Mathematical Finances? No matter how we view them, all these fields are on the border, therefore they will be similar to the lamb which benefits from the “services” of two sheep ...

10. Is Mathematics an aristocratic science? Or snobbisms elevated to art?

Mathematics is a catalyst for a very large number of communication processes among natural and/or social disciplines, arts, engineering sciences. Who could achieve this better than Mathematics and bring Physics and Physical Chemistry towards economic sciences, starting by Bachelier’s model based on random walk and the Brownian processes (via pollen and gases), continuing with the chaos theory and fractals and reaching the basics of Econophysics. The entropies and their generalizations (Shannon, Rényi, Hartley, Tsallis), as well as the informational type of energies have already been carried towards Economic sciences. Another interesting application from Physics is potential regression, concerning which I have written a brief paper. It all starts from the inconvenience of linear regression that is based upon the method of the smallest squares applied to the sum errors of approximating y by f(x). The basic idea is the use of the mechanical principle of balancing forces and minimizing the potential energy of a system of forces, particularly elastic forces perpendicular on the regression straight-line and not parallel to the axis of ordinates, as in the case of classic linear regression. Probably, the viewpoints that draw Mathematics towards a position aspiring to that of reine des sciences do nothing else but create this impression. We, mathematicians, like to believe that the access to Mathematics is sufficiently

---

51 According to [Marcus1987], p. 17.
52 Some also call it Biomathematics.
53 Initially begun around grammars and Chomski’s hierachy, but eventually extended to the Algebra of automata (see [Atanasiu2007]) or probabilistic grammars (see [Jalobeanu2008]).
54 Some also call it Bioinformatics.
55 See [Marcus1987], p. 116.
56 Or the stochastic process “drunkard’s walk”, see [Bachelier1900].
57 See Brown’s experiences, see [Socaciu2009a].
58 See Maxwell’s laws, see [Socaciu2009a].
59 See [Socaciu2009e].
60 See [Shannon1948]; also known as informational enthropy or Shannon-von Neumann enthropy or Boltzmann-Gibbs enthropy or Shannon-von Neumann-Boltzmann-Gibbs enthropy.
61 See [Rényi1960], generalization of Shannon’s enthropy.
62 A particular case of Rényi’s enthropy.
63 See [Tsallis1988], [Havrda1967], [Daroczy1970], generalization of Shannon’s enthropy, also known as Havrda-Charvat-Daroczy-Tsallis enthropy.
64 For example, the Onicescu energy.
65 See [Calude1982], pp. 7-30, on the use of the Onicescu method in achieving hierarchies in the case of multi-criterion optimizations.
66 A drawback that is one of the points investigated by classic Econometry.
67 See [Socaciu2008] and [Socaciu2009f].
68 queen of sciences (in French).
democratic, and the royal cataloging and labeling are only the result of those with other jobs who come into contact with it but who manage to successfully operate with notions, concepts, theories (engineers, physicists, financiers), some of them even on a high level (for example, Albert Einstein), reminding us that Mathematics is an *Ars Magna*.

11. Ways of rephrasing the question we have started from

If I try to think about the denial of Mathematics by some of the economists, a denial which can sometimes be elevated to virtue, or golden rule, I feel I have to rephrase the question we have started from in other ways, such as:

1) How much Mathematics do Economic Sciences need?
2) How much Mathematics do economists need?
3) How much Mathematics does economy need?

I have tried to find a real answer by question my colleagues and students discreetly. I do not think that the publication of unprocessed answers is desirable...

12. Bibliography and references


---

69 However, not with an esoteric meaning, see [Huitin1992].

70 I have used a small letter because I do not mean the science.


