Abstract
In the quest for new, innovative energy sources, an important place is held by renewable energy obtained from biomass. Solid biofuels (pellets and briquettes) obtained from one or more types of biomass are easy to obtain and ready to hand. However, one cannot achieve true technological innovation without performing a modicum of academic research. This paper presents a linear programming tool which can be used to determine the most efficient mixture of biomass in terms of calorific value and with respect to constraints imposed by international standards (noxes and ash content). To this end, a database of raw materials existent in the area was built. The properties of the biofuels were modeled based on the properties of their components, through previously validated mathematical models and formulas.

Keywords: Linear Programming; Mathematical Modeling; Optimization; Biofuels.

1. Introduction
Solid biofuels obtained from biomass represent a cheap, reliable and handy source of energy (REN21). On the other hand, pollution due to a more or less indiscriminate usage of solid fuels, as well as its effects, represents a well-established fact.

While almost any biomass type can be burned in order to obtain energy, there are specific standards to be met in order to classify the biomass as solid biofuel. These quality requirements are stated in the international standards ISO17225 and EnPlus and refer to the origin of the biomass, physical parameters (size, bulk density, calorific value) dust and ash content, additives amount and chemical composition (like N, S, Cl, As, Cd, Pb, Hg, Ni, Zn, etc.).

Acknowledging pollution and its effects led to the development of environmental policies and regulations. Thus, between 2011 and 2013 EU Commission devised a framework concerning the quality of clean air (COM 2013). In this regard, new regulations were adopted in order to limit the effects of industrial pollution on the atmosphere, engaging each EU country to implement air pollution control...
programs and meet the pollution reduction commitments for 2020 and 2030, especially regarding the reduction of dust, sulphur dioxide, nitrogen oxide and other toxic volatile compounds (e.g. chlorine compounds).

Hence, the industry of solid biofuels deals with two major trends: the first, to produce highly efficient biofuels, i.e. to maximize the calorific value of the biofuels and the second to keep the emissions due to the above mentioned biofuels within the limits dictated by the pollution reducing regulations.

From this perspective, a Romanian private microenterprise which produces heating pellets and briquettes faces the same challenges as any of its European counterparts. Pellets are small particles and briquettes are larger blocks, both obtained by compressing solid biomass in a regular form (usually cylindrical). In the following, we refer to a microenterprise which is a spin-out of the Ioan Slavici Foundation for Culture and Education and member of the Cluster for Environment and Renewable Energy Sources WESTTIM (MERWT). As a spin-out, the enterprise has the purpose of commercial valorization of scientific research results, in particular by producing pellets for heating from innovative materials and mixtures, while abiding the EU regulations concerning the air pollution.

The problem of determining a mixture with the maximum possible calorific value while maintaining the concentration of emissions lower than a certain level is a classical linear optimization problem. Previously, we solved this problem using a brute-force algorithm and for mixtures of two components (Maris et al., 2018) and later we analyzed the influence of additives to the two-component mixtures obtained by applying the same brute-force algorithm (Maris, Maris and Tucu, 2018).

While linear optimization is classically used for project selection and production planning, it could be also used to determine optimal solutions for financial investments of business (Chumburidze et al., 2019), for determining product mixtures (Chen et al., 2011) and even food recipes (Ryan et al., 2014 and Quijano-Aviles et al., 2016). A mixed-integer linear programming technique was used in 2017, by Lee and Kim, during the elaboration of a feasibility study for the production of energy from biomass.

However, up to our knowledge, linear programming is yet to be applied to the production of optimized biomass pellets, in the sense mentioned before (highest calorific value possible while emissions do not surpass a certain amount). Thus, the aim of our paper is to present a software solution we devised, which computes a mixture recipe for optimized biomass pellets – whenever this mixture is feasible. The software, which is implemented in C, uses a database which contains the most usual biofuel in the area. In addition to the optimization problem, which is able to process more than two inputs and additive, the software we devised allows the user to add a new material to the database and to use his own standards in computing the solution.

2. Materials and methods
   A. The problem

While the real problem can be somewhat difficult to solve using the classical, time-consuming approach of empirical experimental trials – even using special techniques for experimental design, the linear programming approach offers a solution to the problem in almost no-time and for any combination of biomass inputs.

In real life the usage of more than two components to the biomass mixture may raise practical and logistical issues, as each raw material should be stored in a separate bunker and special equipment should be used in order to feed all the components in the mixer. This issue does however not affect the computational approach.

By solving the real-life problem, we understand to offer an answer, stating either that the combination of inputs does not yield to an output with respect to the constraints, or listing the mass ratio associated to the contribution of each component of the mixture.

From the computational point of view, the problem to be solved is: given \( n \) materials for which the elemental composition (C, H, S, O, N, Cl) resulting ash and calorific value (in MJ/kg) are known, find a mixture of them (i.e. find the mass \( m_i \) of each component \( i \) of the mixture) which maximizes the calorific value, while the resulting concentrations of N, S, Cl and ash remain lower than some limit values imposed by the international standards.
In stating the problem, we should underline that the pelletization is a pure physical process and thus the chemical composition of the resulting pellets is identical to the chemical composition of their components. Moreover, the amount of a certain chemical element in the resulting pellets is the weighted average of that chemical element in the biomass components used for pelletization.

Thus, the problem to be solved may be formulated as:

\[
\begin{align*}
\text{max } Q &= 0.339 C + 1.029 H + 0.109 S - 0.109 O \\
\text{if } C &= \frac{m_1}{m_1 + \cdots + m_n} C_1 + \cdots + \frac{m_n}{m_1 + \cdots + m_n} C_n \\
H &= \frac{m_1}{m_1 + \cdots + m_n} H_1 + \cdots + \frac{m_n}{m_1 + \cdots + m_n} H_n \\
S &= \frac{m_1}{m_1 + \cdots + m_n} S_1 + \cdots + \frac{m_n}{m_1 + \cdots + m_n} S_n \\
O &= \frac{m_1}{m_1 + \cdots + m_n} O_1 + \cdots + \frac{m_n}{m_1 + \cdots + m_n} O_n \\
N &= \frac{m_1}{m_1 + \cdots + m_n} N_1 + \cdots + \frac{m_n}{m_1 + \cdots + m_n} N_n \\
Cl &= \frac{m_1}{m_1 + \cdots + m_n} Cl_1 + \cdots + \frac{m_n}{m_1 + \cdots + m_n} Cl_n \\
\text{ash} &= \frac{m_1}{m_1 + \cdots + m_n} ash_1 + \cdots + \frac{m_n}{m_1 + \cdots + m_n} ash_n \leq ash_{max} \\
0 &\leq N \leq N_{max} \\
0 &\leq S \leq S_{max} \\
0 &\leq Cl \leq Cl_{max} \\
1 &\leq m_1, \ldots, m_n \leq M_{max}
\end{align*}
\]

One should note that the formula for Q is the Mendeleev formula.

A mixture with respect to some standard may or may not contain additives. From a practical point of view, an additive is a substance which could be added in small amounts to the mixture in order to improve its quality or help preserve it. From a mathematical and programming point of view, the additive, added in its maximum value, contributes to the modification of the standard limits.

The relevant values for calorific value Q, N, S, Cl, ash and additives, according to the corresponding standards, are presented in Table 1.

**Table 1. Relevant limit values imposed by ISO 17225 and EnPlus**

<table>
<thead>
<tr>
<th>Standard name</th>
<th>Standard details (materials and usage)</th>
<th>Relevant values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Q (MJ/kg)</td>
</tr>
<tr>
<td>EnPlus A1</td>
<td>Stemwood Chemically untreated wood residues Non-industrial usage</td>
<td>16,56</td>
</tr>
<tr>
<td>EnPlus A2</td>
<td>Whole trees without roots Stemwood Logging residues Chemically untreated wood residues Non-industrial usage</td>
<td>16,56</td>
</tr>
<tr>
<td>EnPlus B</td>
<td>Forest, plantation and other virgin wood By-products and residues from wood processing industry Chemically untreated used wood Non-industrial usage</td>
<td>16,56</td>
</tr>
<tr>
<td>I1</td>
<td>Forest, plantation and other virgin wood</td>
<td>16,50</td>
</tr>
</tbody>
</table>
Problem (1) will be written in a standard form, before being solved numerically.

**B. The database**

Linear programming can be used as a fast solving method for problem (1) provided that a database containing materials and at least information about their elemental composition is available.

For the purposes of our research, a database containing the most common materials available for pelletizing in the area was constructed. The data was either measured, or collected from literature (e.g., Krajnc, 2015, Garcia et al., 2014). For the moment, the database has 76 entries, but it can be extended. While it is customary to add oil as additive in the pelletization process, for the moment we did not consider this option and we did not add any liquid biofuel to our database.

A sample of this database is presented in table 2. While the price is not very important at this moment, it was set to 0 whenever we could not find sufficient data to compute it.

<table>
<thead>
<tr>
<th>Material</th>
<th>C (w-%)</th>
<th>H (w-%)</th>
<th>O (w-%)</th>
<th>N (w-%)</th>
<th>S (w-%)</th>
<th>Cl (w-%)</th>
<th>Price (lei/kg)</th>
<th>Q (MJ/kg)</th>
<th>ash (w-%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fir</td>
<td>51,23</td>
<td>5,98</td>
<td>42,10</td>
<td>0,06</td>
<td>0,03</td>
<td>0,19</td>
<td>0,04</td>
<td>19,19</td>
<td>0,41</td>
</tr>
<tr>
<td>Hemp (strains)</td>
<td>46,10</td>
<td>5,90</td>
<td>42,50</td>
<td>0,74</td>
<td>0,10</td>
<td>0,20</td>
<td>0,00</td>
<td>17,00</td>
<td>4,80</td>
</tr>
<tr>
<td>Wood charcoal</td>
<td>79,34</td>
<td>2,74</td>
<td>16,97</td>
<td>0,65</td>
<td>0,30</td>
<td>0,00</td>
<td>4,50</td>
<td>29,71</td>
<td>5,90</td>
</tr>
<tr>
<td>Spruce (with bark)</td>
<td>49,80</td>
<td>6,30</td>
<td>43,20</td>
<td>0,13</td>
<td>0,02</td>
<td>0,01</td>
<td>0,04</td>
<td>18,80</td>
<td>0,60</td>
</tr>
<tr>
<td>Nectarines (seeds)</td>
<td>48,57</td>
<td>6,22</td>
<td>44,48</td>
<td>0,50</td>
<td>0,23</td>
<td>0,00</td>
<td>0,00</td>
<td>19,56</td>
<td>1,10</td>
</tr>
<tr>
<td>Barley (straws)</td>
<td>47,50</td>
<td>5,80</td>
<td>41,40</td>
<td>0,46</td>
<td>0,09</td>
<td>0,40</td>
<td>0,05</td>
<td>17,50</td>
<td>4,80</td>
</tr>
<tr>
<td>Poplar</td>
<td>47,50</td>
<td>6,20</td>
<td>44,10</td>
<td>0,42</td>
<td>0,03</td>
<td>0,00</td>
<td>0,04</td>
<td>18,50</td>
<td>1,80</td>
</tr>
</tbody>
</table>
C. The Simplex algorithm

In order to solve problem (1), we use the Simplex algorithm, as described by Ferguson et al, 2000.

Consider a linear optimization problem of the form
\[
\begin{align*}
\max f &= \bar{c}^s \cdot x^s \\
x^B + S \cdot x^s &= \bar{b} \\
x^B &\geq 0 \\
x^s &\geq 0
\end{align*}
\]
(2)

Step 1. If all the components of the vector \(\bar{c}^s\) are non-negatives, then the current basic solution is the optimal solution and the algorithm stops.

Step 2. Choose the non-basic index \(k\) such as
\[
\bar{c}_k = \min \bar{c}_j (3)
\]

Step 3. If \(\bar{a}_{ik} \leq 0\) for all the basic indexes \(i\) (i.e., all the elements of \(\bar{A}^k\) are negative or zero), then the problem (2) has an infinite optimum and the algorithm stops.

Step 4. Find the basic index \(r\) such that
\[
\frac{\bar{b}_r}{\bar{a}_{rk}} = \min \left\{ \frac{\bar{b}_i}{\bar{a}_{ik}} \mid i \text{ basic index such that } \bar{a}_{ik} > 0 \right\} (4)
\]

Step 5. Replace in base B column \(A'\) with \(A^k\), obtaining a new basis, \(B'\). Build the explicit form of the problem (2) with respect to the new basis and start again from step 1.

3. Results

A. Mathematical formulation

After reducing the variables, problem (1) can be stated as
\[
\begin{align*}
\max Q &= x_1 Q_1 + \ldots + x_n Q_n \\
\text{if} &
\begin{cases}
x_1 S_1 + \ldots + x_n S_n \leq S_{\max} \\
x_1 N_1 + \ldots + x_n N_n \leq N_{\max} \\
x_1 C_l + \ldots + x_n C_{nl} \leq C_{l_{\max}} \\
0 \leq x_i \leq 1, \ 1 \leq i \leq n
\end{cases}
\end{align*}
\]
(5)

where
\[
Q_i = 0.339 C_i + 1.029 H_i + 0.109 S_i - 0.109 O_i, \ 1 \leq i \leq n
\]
\[
x_i = \frac{m_i}{m_1 + \ldots + m_n}, \ 1 \leq i \leq n
\]
(6)

In this new form of problem (2), the variables \(x_i\) represent the percentage ratio of the material \(i\) in the mixture. Thus, problem (5) has a solution if and only if
\[
x_1 + \ldots + x_n = 1. \ (7)
\]
If additives are present, problem (5) is rewritten as
\[
\begin{align*}
\text{max } Q &= x_1 Q_1 + \ldots + x_n Q_n + x_{ADD} Q_{ADD} \\
\text{if } &\quad x_1 S_1 + \ldots + x_n S_n \leq S_{max} - x_{ADD} S_{ADD} \\
&\quad x_1 N_1 + \ldots + x_n N_n \leq N_{max} - x_{ADD} N_{ADD} \\
&\quad x_1 Cl_1 + \ldots + x_n Cl_n \leq Cl_{max} - x_{ADD} Cl_{ADD} \\
&\quad x_1 ash_1 + \ldots + x_n ash_n \leq ash_{max} - x_{ADD} ash_{ADD} \\
0 &\leq x_i \leq 1, \ 1 \leq i \leq n
\end{align*}
\]

and the percentage ratio condition (7) becomes
\[
x_1 + \ldots + x_n = 1 - x_{ADD}. (7')
\]

We admit that the value for \(x_{ADD}\) is a constant depending on the chosen standard, being the maximum allowed by it.

**B. Software solution**

The software solution we created consists in a main menu which offers more options: analyze the standard limitations, find an optimal recipe based on the database inputs, add a new material to the database and, of course, exit the program. After each response of the program, the main menu is displayed and the user can choose different options, over and over again, until a (more or less) satisfying response is found before exiting the program.

If the user chooses to consult the standards, the limit values for N, S, Cl and ash will be displayed for the predefined (international) standards. However, it should be stated that the user is allowed to use his own standard, in determining a recipe for a new mixture.

If the user chooses to add a new material in the database, a warning is displayed – that is, it is imperative to know the percentage ratio values for N, S, Cl, ash and for Q in MJ/kg.

The main feature of the program is the determination of an optimal recipe. If this option is chosen, the database is read and the names of the materials (and their corresponding positions in the database) are displayed. The user should enter the numbers corresponding to the materials desired, ending with 0 when no other materials are desired for the current recipe.

Next step is to choose an additive. If so chosen, the additive will influence the values of the standard limitations (as shown in figure 1). Thus, the program will display the relevant values associated to the chosen additive.

Further, the user should choose the standard to be used during the computations. One option is to add his own standard (own limit values for N, S, Cl and ash).

If an additive was already chosen, the current values of the standard are modified accordingly and displayed.

Finally, whether using an additive or not and within the limitations of the chosen standard, a recipe is computed using the simplex algorithm (figure 2).
Figure 1. Screen capture of the program with a recipe computed (additive influence and positive response)

The structure of the main function, translated into English, is presented below, followed by a brief description of the functions used:

```c
void main() {
    int alegered; // Variable to store the user's choice
    goto e200; // Go to label e200
    printf("Choose an option: \n
    1 — See the standards \n
    2 — Find a mixture recipe \n
    3 — Add a new material \n
    0 — Exit the program\n") ;
    scanf("%d", &alegere) ;
    switch(alegere){
        case 1:
            Consultastd() ;
            goto e200 ;
            break ;
        case 2:
            Intrari() ;
            Standarde() ;
            Data() ;
            Simplex() ;
            Results() ;
            goto e200 ;
            break ;
    }
}
```

- **Consultastd()**: Allows the user to view the standard mixture.
- **Intrari()**: Allows the user to input data such as material components and their properties.
- **Standarde()**: Displays the standard mixture details.
- **Data()**: Handles the input of data for the mixture components.
- **Simplex()**: Uses the Simplex method to optimize the mixture.
- **Results()**: Outputs the results of the optimized mixture.

The program provides a user-friendly interface to select options such as viewing standards, finding a mixture recipe, adding new materials, or exiting the program. The structure of the main function is clear and organized, allowing for easy modification and extension.
case 3:
Aduaugamat();
goto e200;
break;
case 0:
printf("Goodbye!\n"); break;
}

Figure 2. Screen capture of the program with a recipe computed (choosing the materials, no additive and positive response)

The function Consultastd() prints the limit values for the N, S, Cl, ash and additive for each of the standards in use.

The function Intrari() opens the file containing the database, prints the name of the materials from the database and, if the user chooses to use the material i, stores the corresponding values for C, H, O, N, S, Cl, price, Q and ash. The user can choose as many components as desired. If the user choses to
use an additive by entering the number corresponding to it in the database, then the function stores the corresponding database values. In the end, the file containing the database is closed.

The function Standarde() asks the user about the standard values to be used during the computations. The user can choose a predefined standard or can choose to input and use his own standard. If previously an additive was chosen, the standards are modified accordingly and the new values are displayed.

The function Data() builds the linear program corresponding to the problem (5') and additional constraint (7') based on the data selected by the user through the functions Intrari() and Standarde().

The function Simplex() applies the simplex algorithm to the linear program built by the function Data(), by calling, in this order, the functions Pivot(), Formula() and Optimize(). The functionPivot() finds the pivot \( a_{rk} \) from the Step 4 of the algorithm. The functionFormula() computes the new basis in the simplex algorithm, based on the pivot previously computed. The functionOptimize() establishes if the solution computed by the function Formula() is optimal or not.

The function Results() prints the output of the algorithm: if the solution is not optimal or does not sum to 100% (thus does not fulfill (7')) returns a negative response; else, prints the percentage composition of the mixture, the price of each material and the resulting calorific value (in MJ/kg).

The function Adaugamat() allows the user to add a new material to the database. In order to add a new material, the user should be able to provide the values considered essential from the computational point of view, i.e. N, S, Cl, ash and Q.

The results in figures 1 and 2 can be summarized as follows:

- The user tries to find a mixture recipe containing wood chips, oak branches and chips, and vine branches, within the limits of the standard MBP B.
- The first approach uses lignin as additive; the second approach uses no additive.
- The optimal recipe using additives contains 47,20% oak branches and chips; 47,80% vine branches; 5% additive (lignin) and no wood chips, at a calorific value of 16,24 MJ/kg
- The optimal recipe in absence of additives contains 15,79% wood chips; 15,79% oak branches and chips; 68,42% vine branches, at a calorific value of 16,46 MJ/kg.

One should note that, while the user can choose virtually as many components as desired for a mixture, the software may not retain all of them for the optimal recipe. While from a computational point of view the number of components should be less than a maximum value (in our case we chose 20), the number of components chosen by the user is limited solely by the design of the pellet mill. Also, one should note than not always the additive contributes to a higher yielding mixture in terms of calorific value.

For now, the software does not list the chemical composition of the mixture in terms of N, S, Cl and the ash content, as from a computational point of view it is not important. However, should an user decide to sell the pellets, these quantities should be computed and used on the product label, and the production price should be computed as well.

4. Conclusions

There are many ways to solve the practical problem of determining a mixture of solid biofuels which yields the highest calorific value while keeping the percentage of critical elements (N, S, Cl) and ash content below some limit values. The traditional method, through experimental trials and statistical data processing is time and resources consuming, even using techniques for designing the experiment.

In order to overcome these aspects, we propose a new approach, based on linear programming. This solution is fast, cheap and user-friendly and offers apriori data on how the field experiments should be conducted in order to obtain pellets with a certain chemical composition and calorific value.

Further improvements of our work should add the chemical composition of the mixture to the final results; should add a visual interface; and should be able to determine the best choices for two component mixtures (with or without additive) given only one component input.
References


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